Dynamic Asset Sales with a Feedback Effect

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I analyze a dynamic model of over-the-counter asset sales in which the seller receives stock-sensitive compensation, and the transaction conveys information about the firm's value. I examine how the market's response to an asset sale feeds back to the seller's decision on the timing and the sale price and analyze the unique pattern of stock prices before and after the sale. The implications of bargaining power, inventories, gains from synergy, and the introduction of a vesting period are discussed. The model sheds light on observed properties of corporate sell-offs and explains market dry-ups during downturn periods. (*JEL* G12, G14, D82, C73)

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Real actions made by firms are often observable and as such enable market participants to learn new information about the value of those firms. At the same time, managers' compensation in public firms is often tied to the stock price and is therefore affected by the release of such information.[1] Therefore, real activity within public firms may not only affect their market value but also may be affected by it, leading to price patterns and activity patterns that cannot be explained separately.

This paper aims to analyze such a joint dynamic of real activity and stock prices. Specifically, I examine a dynamic discrete-time model of over-the-counter asset sales, where an informed seller aims to sell an asset to a stream of short-term buyers, and matching frictions exist. The paper diverges from the

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[1] Empirical evidence shows that managers take into account the information that their actions disclose to the market. Agarwal and Kolev (2013) show that managers of public firms prefer to conduct mass layoffs in a recession month, when layoffs signal less about the firm's relative condition. Several papers (e.g., Sekine et al. 2003; Peek and Rosengren 2005) find that banks in Japan continued to lend to severely impaired borrowers during the 1990s, to avoid realizing losses on their own balance sheets.

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previous literature by assuming that the seller cares not only about the value of her asset but also about *the public beliefs regarding this value* and that buyers acquire private information about the value of the asset prior to a sale. This combination of assumptions fits many situations in which public corporations trade in assets of significant value. In such cases, managers may care about the market value of their firm (which represents the public belief about the value of the firm's assets), either because of possible liquidation concerns by shareholders or because of stock-based compensation (for more details, see Section 1). Moreover, when the traded assets are nonstandard, potential buyers have an incentive to perform due diligence, thus making them more informed than investors at the time of a sale.

The model leads to some new results on asset sales and stock price patterns. In contrast to standard models with adverse selection, where only low-quality assets ("lemons") are sold in each period, in the present model only sellers with high-quality assets choose to sell. The market response to a sale is, on average, positive, while an event of "no-sale" signals low quality. Thus, and in contrast to most dynamic "lemon market" models, market values and sale prices decrease over time until a sale occurs. Moreover, the model generates value-destroying sales, where the seller accepts a price that is less than the value of holding the asset. Sales occur even when the buyer and the seller have the same valuation for the asset. Indeed, even when the buyer values the asset more than the seller, value-destroying sales may take place for high-quality assets.

To see the intuition behind these results, note that a sale price reflects the information obtained by the buyer. Thus, an asset sale reveals new information to the market and affects the stock price, which, in turn, affects the seller's payoff.[2] Because of information asymmetry, high-quality asset is undervalued, and the seller wants to sell such an asset because a sale releases positive information and increases the stock price. Such a seller can gain even from a value-destroying sale due to the information externality. A seller with a low-quality asset, on the other hand, owns an overvalued asset and prefers not to sell. Investors gradually infer that the asset has low quality if they do not see a sale for a long period of time, and hence the market value of firms that do not sell decreases over time. An asset that was previously overvalued become undervalued, and that gives the seller an incentive to sell in subsequent periods.

The model in this paper contributes to the literature on dynamic "lemon market" models used to describe over-the-counter markets. This is not the only model where the quality of sold assets decreases over time. First, Taylor (1999) and Kaya and Kim (2018) show that when buyers observe noisy signals, then no-sale may also be a negative signal. Their results rely on the imprecision of the signals, and arise in cases where the seller's sale decision is independent

[2] Malmendier et al. (2016) analyze failed takeovers, and find evidence that a bid announcement reveals price-related information on the target. Though their context is somewhat different, their finding supports the claim that potential buyers acquire superior information, and that the release of such information affects pricing.

of her type and so a sale relies only on the buyer's signal (for more details, see the literature review below). Second, a decreasing price trend may also be the result of screening: a seller who does not know the valuation of the buyers may at first ask for a high price and then slowly lower this price over time.[3] By contrast to these two types of models, in the present model the seller sells only if her assets are above some value threshold and, moreover, value-destroying sales may take place. These features lead to additional predictions about trading volume and endogenous choice of quantity.

Standard "lemon market" models fit situations in which the seller directly owns and utilizes the asset.[4] The present model, as mentioned above, better fits situations in which public corporations trade in assets of significant value. Below, I will consider two specific examples: corporate sell-offs and sales of (portfolios of) structured financial assets. In the context of corporate sell-offs, it is shown that positive announcement returns, interpreted in the empirical literature as an evidence for value-creating sales, may be observed even following value-destroying sales. The model's prediction that no-sales are bad signals about the value of the firm also finds some supportive evidence in the data. I also outline a testable prediction on the pattern of returns after a firm declares a plan to divest an operation.

In addition, it is shown that feedback effects may have a large effect on volume in markets for structured financial assets. I first show that the seller is less willing to sell (sells only if value is above a higher threshold) when less buyers are available, and that this effect is amplified when the seller has a large leftover inventory of similar assets to the ones that are sold. This is because greater matching frictions and higher inventories increases future payoffs, and hence decreases the cost of waiting. Thus, the model predicts that the feedback effect exaggerates the impact of demand shocks on volume: a modest decrease in demand results in a large decrease in volume (a "dry-up"), because of changes in the seller's behavior. An example is the dry-up of markets for collateralized debt obligations (CDOs) and mortgage-backed securities (MBSs) during the financial crisis of 2008, when financial institutions with large inventories halted trade. It was suggested that banks avoided trading in structured assets during the crisis because trade would have forced them to lower the value of their inventory, which could have led to insolvency.[5] While banks operate under a regulation that demands mark-to-market pricing, the model shows that the

[3] I thank an anonymous referee for this useful example.

[4] Some papers, such as Kremer and Skrzypacz (2007) and Daley and Green (2012), assume that the seller has an option to sell the asset at a later time after its value is disclosed. This assumption generates the same result as when one assumes that the seller obtains a direct utility from ownership.

[5] See, for example, the op-ed by Kenneth E. Scott and John B. Taylor published in the *Wall Street Journal* on July 21, 2009. The authors write: "In September 2008 credit spreads skyrocketed and credit markets froze. By then it was clear that the problem was not liquidity, but rather the insolvency risks of counterparties with large holdings of toxic assets on their books."

same effect may occur when assets are priced using rational expectations. This feedback effect has consequences for the optimal regulation of these markets.

The paper presents additional results that arise because of the feedback between asset sales and the financial market. When the seller can choose the quantity of sold assets, the model predicts that assets of higher value are associated with a smaller deal and higher leftover inventory. This result, previously presented in the literature as a result of signaling (Leland and Pyle 1977; Demarzo and Duffie 1999), arises in this framework when the buyer is informed, and relies on sufficient bargaining power by the buyer. It is also shown that the results of the model are robust to the case in which the buyer also has stock-sensitive payoff and thus cares about the market response to a sale. Finally, the introduction of a partial vesting period does not alter the equilibrium, and as long as the seller cares about the stock price in some periods, value-decreasing sales may take place in all periods.

A series of papers (Janssen and Roy 2002; Kremer and Skrzypacz 2007; Daley and Green 2012; Fuchs and Skrzypacz 2015; Dilmé and Li 2016) analyze dynamic versions of the "lemon market" model (Akerlof 1970). In these papers, sellers with higher values have a higher reservation utility, and can thus signal their type to buyers by waiting. As a result, sold assets are increasing, on average, in price and quality over time. In this paper, buyers are informed, so no such signaling occurs, and sale prices reflect the true value of the asset.[6]

The seller, however, does care about the beliefs of the market, and her decision to sell depends on the information that a sale discloses to the market. Therefore, the paper is much related to the theoretical literature on voluntary disclosure. Dye (1985) Equivalently, the present model has random matching between the seller and the buyer. Guttman et al. (2014) present a dynamic disclosure model, in which an informed manager can choose to disclose in each of the periods. In contrast to their paper, this paper assumes that selling/disclosure opportunities are short lived.[7]

Kaya and Kim (2018) and Taylor (1999) analyze dynamic models of an asset sale where buyers observe noisy signals. In their models, as in mine, prices and qualities of sold assets may be decreasing over time. Such trend occurs when the seller is willing to sell only for a high price, even when she owns an asset of low value. Thus, sales take place only following a buyer's high signal, and no-sale is associated with low signals, which are more frequently generated by low types. The same effect is observed in Martel (2018), who presents a model where sellers are uninformed, and only buyers observe noisy signals. By contrast, in this paper, buyers observe perfect signals, and, therefore, seller

6 See also Guerrieri and Shimer (2014) and Zhu (2012) for other dynamic over-the-counter models where qualities and prices change over time. These two papers also differ from the present one in that traders are not compensated according to the market's response.

7 Acharya et al. (2011) present a dynamic disclosure model where the timing of disclosure is affected by an exogenous public signal. Einhorn and Ziv (2008) and Beyer and Dye (2012) present disclosure models where the manager's asset value changes in each period and focus on reputational effects that do not appear in this paper.

types do not pool; in each period high types choose to sell, whereas low types choose not to, and the set of selling types changes over time. The seller's strategic considerations also allow for value-destroying sales and patterns of optimal retention that do not appear in the previous literature. An alternative interpretation of the present model, which appears in Remark 1, assumes a value-maximizing seller that faces a population of buyers, some informed and some uninformed. Although this interpretation generates a decreasing price trend, it does not predict value-destroying sales, which can only occur because of a feedback effect.

One interpretation of the model is that the seller is a delegated manager with stock-based-compensation. An established theoretical literature tackles the actions and possible inefficiencies of managers with stock-based compensation (Fishman and Hagerty 1989, Stein 1989, Paul 1992; for a recent survey, see Bond et al. 2012, section 3). This paper contributes to this literature by showing that delegated managers' real actions may be affected by the information that such actions reveal. A similar example can be found in Benmelech et al. (2010), where managers use a suboptimal investment policy to conceal the fact that a firm's growth opportunities have declined.

Finally, the negative effect of inventories on trade volume in over-the-counter markets is also explored in Milbradt (2012) and Bond and Leitner (2015). Both papers deal with firms that have asset-backed loans. These firms forgo profitable transactions when such transactions decrease the value of their inventory, in order to avoid a violation of their capital constraint. By contrast, the result here does not rely on the existence of leverage, but rather demands a certain negative demand shock to generate the dry-up. An inventory simply amplifies a contraction in volume that will take place in any case. Thus, the papers can be seen as complementary, each describing a different set of conditions for a market dry-up. Section 5.2 discusses these papers in more detail.

1. A Model

A single firm owns an asset with a fundamental value of v. The value is randomly drawn at the beginning of the game and remains constant. The initial distribution of v can be represented by a cumulative distribution function F_0 with a partial distribution function f_0 that is nonatomic and has full support over a (possibly unbounded) interval $\left(\underline{V},\overline{V}\right)$, where $\underline{V}<\overline{V}$, $\underline{V}\in\mathbb{R}\cup\{-\infty\}$, and $\overline{V}\in\mathbb{R}\cup\{\infty\}$.[8]

At the beginning of the game, v is known only to the seller and is referred to as the seller's "type." The game comprises two selling periods and three dates, $t\in\{0,1,2\}$. A previous version of this paper, available on the author's Web site, generates the same results in a general finite-horizon model with T

[8] The assumptions on f_0 are for mathematical ease. Most results hold qualitatively even without them. The interval of v is assumed open (and hence limits are imposed), but results hold when the interval is bounded and $v=\underline{V}>-\infty$ and/or $v=\overline{V}<\infty$.

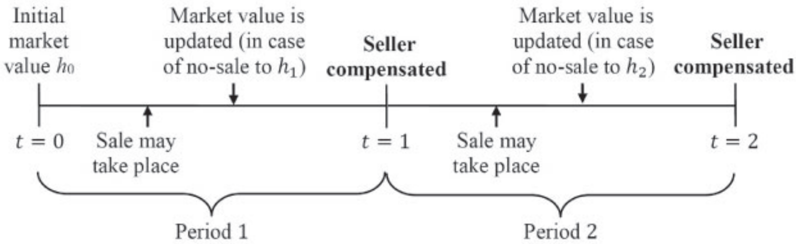

Figure 1
Time line of the game

selling periods. The time line of the game, presented in Figure 1, is described in detail below.

1.1 Periodic sell offers

In each of the two periods, a potential buyer is matched with the seller with probability q and negotiates a price offer with the seller.[9] The offer is short term ("explosive"), and if there is no sale the buyer leaves to make an alternative investment. With probability $1-q$ nothing happens during the period.[10]

It is assumed that the negotiations between the buyer and the seller are private and their existence is kept secret in the case of failure. Indeed, nondisclosure agreements (NDAs), which are frequently used in corporate negotiations, usually include a nondisclosure of discussions clause to protect the confidentiality of the negotiations. As will become clearer later, the key assumption here is that in a period where no sale takes place, the market cannot tell whether this is because of exogenous reasons (no match) or because of the seller's decision not to sell. Thus, for example, the same results are obtained if buyers arrive in all periods, but with probability $1-q$ negotiations break down for reasons that are independent of the asset's value (e.g., legal reasons, regulatory reasons, or differences in corporate culture, which are discovered in the course of the negotiations). Same qualitative results are also obtained if information about failed negotiations might "leak" with some probability. If, however, a seller's decision not to sell always becomes public, then in equilibrium a sale will always take place when a seller and a buyer meet, unconditional on the asset's value. This is the "unraveling result" that was first presented by Milgrom (1981) and Grossman (1981).

9 The results of the model remain qualitatively unchanged even if more than one buyer may arrive in a given period, as long as there is a positive probability that no buyer arrives.

10 In reality, firms do not always publicly announce their intention to sell an asset. Thus, it may take some time for potential buyers to learn that an asset is for sale. A possible way to incorporate this idea into the model is to assume that the matching probability is time dependent and increasing over time. The results will be qualitatively the same.

1.2 Value of ownership

The fundamental value v is the net present value of the asset's future cash flow. This cash flow is perpetual and comprises expectedly equal payments, and thus v is independent of time. In the baseline model, the cash flow and valuation are the same for both the seller and the potential buyers. Section 4 discusses the case in which buyers utilize the asset better and therefore value it more than the seller.

1.3 Seller's preferences

In each period, the seller enjoys a payoff that depends not only on the utilization of the asset (the generated cash flow) but also on its market value (the public belief about this cash flow). For simplicity it is assumed that the seller's periodic payoff before a sale is linear and equals

$$\chi \cdot h_t + (1-\chi)v,$$

where h_t is the market value of the firm and $\chi \in (0,1]$. The parameter χ is the relative importance of the stock price to the payoff of the seller. After a sale, the firm's asset is simply the cash it obtained, p. Because sale prices are observable, the firm's market value after a sale is also p, and the payoff of the seller is independent of χ. In each period of the game t, the seller maximizes *the discounted sum of her expected periodic payoffs* until the end of the game.[11]

The seller's preferences may represent the preferences of the firm's liquidity-constrained shareholders. If the shareholders are vulnerable to liquidity shocks that may induce them to sell some or all their stake in the firm, then they should be concerned not only with the future cash flow of the firm but also with current and future market values.

Alternatively, the seller's preferences may represent the preferences of a firm's delegated manager, who is in charge of the decision to sell. Even if all shareholders only care about the fundamental value of the asset, the manager's payoff may still be sensitive to the stock price. One reason for a manager to be concerned about market value is career concerns (Holmström 1999): the manager has an incentive to influence the stock price if it serves as a proxy for her unobservable ability to govern other operations within the firm (which, for brevity, are left unmodeled).

Another possible reason for a manager to be concerned about market value is stock-based compensation, which is common in public firms. In the present context, shareholders who value an asset according to its fundamental value do not have an incentive to give the manager stock-based compensation when it comes to asset sales. However, stock-based compensation may still be part

11 The payoff assumed here is linear in the stock price, both within and between periods. One can think of other, nonlinear, stock-sensitive contracts (e.g., an option-based contract). Nonlinearity creates additional incentives for the seller to sell or not sell the asset. It also means that the seller is no longer indifferent to how her payment is distributed over time. I abstract away such incentives in this paper, to focuson the information channel.

of an optimal contract if the manager has additional responsibilities within the firm (which, again, are left unmodeled for brevity). In such a case, shareholders may compensate the manager with stocks in order to incentivize her to exert costly effort in other operations where her action is unobserved (Harris and Raviv 1979; Holmström 1979).

1.4 Sale prices

I assume that each potential buyer, as part of the negotiations with the seller, fully observes v, and negotiates a price accordingly. The buyer's outside option (his payoff in case he does not buy) is normalized to zero.[12] The negotiated price is a function of the seller's reservation price, denoted by $u_t^{NS}(v)$ and defined below, and the fundamental value, which is the maximal value the buyer is willing to pay. Formally,

$$p\left(v, u_t^{NS}(v)\right) \equiv (1-\lambda) \cdot u_t^{NS}(v) + \lambda \cdot v, \tag{1}$$

where $\lambda \in [0,1]$ is the relative bargaining power of the seller.[13] For the sake of exposition, and without loss of generality, consider the following bargaining procedure: with probability λ the seller makes a take-it-or-leave-it offer and thus $p = v$, while with probability $1-\lambda$ the buyer makes such an offer and therefore $p = u_t^{NS}(v)$. Under this interpretation, which I follow below, λ is the probability of the seller to obtain the entire surplus from trade.

The seller's reservation price (or outside option) in period t is derived from the (endogenous) expected discounted payoff she expects in case she chooses not to sell, denoted by $U_t^{NS}(v)$ (NS stands for "no-sale"). In the last period, $u_2^{NS}(v) \equiv U_2^{NS}(v)$ as it is simply the payoff from not selling. The outside option in period 1, $u_1^{NS}(v)$, is defined such that, if the seller receives u_1^{NS} in each of the two periods, then her discounted payoff is U_t^{NS}, that is, $u_1^{NS}(v) \equiv \frac{U_1^{NS}(v)}{1+\beta}$, where β is the seller's discount factor.

Sale prices are observed by the market. Thus, if the asset is sold in period t for a price p, then from that point on the firm's net present value and its market value are simply the firm's cash holdings, which are p. Thus, the period payoff of the manager from that period onward is p. Therefore, the seller will sell in period $t \in \{1,2\}$ if and only if p is above $u_t^{NS}(v)$. Note that $u_t^{NS}(v)$ is a function of future prices and therefore is also a function of λ.

[12] I assume that all information obtained by the buyer is confidential under a nondisclosure agreement (NDA), and so the buyer is forbidden from revealing or selling it, or from taking a position on the firm's shares based on such information. Thus, if no sale occurs, this information is not disclosed.

[13] A theoretical motivation for Equation (1) is the asymmetric Nash bargaining problem (Kalai 1977). The price function (1) maximizes the Nash product $\left(p-u_t^{NS}\right)^{\lambda}(v-p)^{1-\lambda}$. Binmore et al. (1986) show that this solution approximates the equilibrium outcome of an alternating offers model of bargaining, in which parties have different time preferences or different estimates about the risk of a negotiation breakdown. Another interpretation of λ is the scarceness of the asset; Ahern (2012) presents empirical evidence that in mergers the relative scarceness of the target determines the division of gains between the seller and the buyer.

The price function of (1) implies that trade always takes place when $u_t^{\mathrm{NS}}(v) \leq v$, that is, when the trading partners can benefit from the trade. This is in line with the idea of bargaining under complete information.[14] The price, however, is below the fundamental value even when trade takes place: $p \leq v$, with strict inequality for $\lambda < 1$. Thus, a sale actually decreases the (net present) value of the firm. The sale will take place nevertheless, because the seller cares not only about the fundamental value of the firm but also about its perceived value, that is, its market value.

1.5 Market values

The initial market value of the firm is denoted by h_0. The market value of a firm that hasn't sold the asset until the end of period $t \in \{1,2\}$ is denoted by h_t (h_t^* denotes equilibrium market values). I assume risk-neutral pricing, where the market value of the firm equals the expected value of owning the firm at the end of the game. Thus, the market value h_t considers not only the expected value given that the firm hasn't yet sold the asset, $E_t[v]$, but also the possibility that the asset may be sold in future periods for prices that differ from its value. In each period, if only some types of the seller want to sell, then an event of no-sale reveals information about the asset's value and affects the firm's market value. After the sale the firm's only asset is the cash obtained from the sale, p, and its market value is also p (this is no longer true if only part of the asset is sold; see Section 5.2).

1.6 Equilibrium definition

I find a perfect Bayesian equilibrium of the game. Such an equilibrium comprises, in each period $t \in \{1,2\}$, the following: (1) a strategy of the seller to sell or not to sell ($\{\mathrm{S},\mathrm{NS}\}$) given her type, (2) a price p_t that depends (through u_t^{NS}) on current and future market values, and (3) a public belief about the fundamental value in the case of no-sale in this period, represented by the partial and cumulative distribution functions $f_t(v)$ and $F_t(v)$, respectively. A Bayesian equilibrium demands that the belief be updated using Bayes rule given the strategy of the firm in that period. The market value in each period and the outside option, u_t^{NS}, are fully determined by the above.

1.7 Equilibrium notation

As will become clear in the next two sections, the equilibrium of the game has a threshold selling strategy; that is, in each period the seller sells if she receives an offer and her asset's value (type) is above some threshold. Henceforth, the

[14] The idea is that if bargaining is modeled as a sequential game, a price $p < u_t^{\mathrm{NS}}(v) < v$ cannot be part of a subgame perfect equilibrium, because the buyer has an incentive to offer a higher price $p' \in \left(u_t^{\mathrm{NS}}(v), v\right)$ that will be accepted and will induce a higher payoff to both parties. Although the model does not include an explicit description of the bargaining process, it preserves this idea.

equilibrium threshold type in period $t\in\{1,2\}$ is denoted by v_t^*. $F_1(v)$ is then a function of the prior F_0 and v_1^*, whereas $F_2(v)$ is a function of F_0, v_1^*, and v_2^*. $E_0[v]$ denotes the expected fundamental at the beginning of the game and $E_t[v]\left(v_t^*,q\right)\equiv E_{F_t\left(v;v_t^*,q\right)}[v]$ denotes the expected fundamental at the end of period $t\in\{1,2\}$ given that the firm uses a threshold strategy v_t^* and that there was no-sale until the end of that period. The equilibrium market values for a firm that hasn't sold are denoted by h_t^*, $t\in\{0,1,2\}$. A market value h_t is affected by sales and sale prices that occur both before and after period t, and thus all market values are functions of v_1^* and v_2^*.

Remark 1. (an alternative interpretation). Before I turn to the equilibrium, I present an additional possible interpretation of the model under which the seller's preferences are not sensitive to the market value, but, instead, some buyers are informed, and some are uninformed.[15] Consider a model where the seller manufactures and sells one unit *each period*. The seller's type determines the quality of all units, and her reservation price is always zero. The seller is matched with an informed buyer with probability q, but uninformed buyers are always available, and cannot observe whether the seller has an offer from an informed buyer. The seller observes whether the buyer is informed and posts a sale price accordingly. Such a model is equivalent to the model presented here with $\chi=1$ and $\lambda=1$. Sales to uninformed buyers are made at a price of $E_t[v]$ and play the same role that the stock-sensitive payoff plays in the original model (note that when $\lambda=1$ then $h_t^*=E_t[v]$). When the seller has the option to sell to an informed buyer, she will exercise this option only if her asset's quality is above a given threshold. Under this interpretation, sales are value enhancing, like in standard "lemon market" examples, and the results on endogenous retention (see Section 6.1) no longer hold.

2. Equilibrium of a Static Benchmark

I start by analyzing the equilibrium in a one-shot version of the model, which is equal to the game's second period. The seller chooses to sell if and only if $p_2(v)\geq u_2^{\mathrm{NS}}(v)$. Substituting (1), this condition becomes $v\geq u_2^{\mathrm{NS}}(v)$. Finally, in a static model $u_2^{\mathrm{NS}}(v)=\chi h_2+(1-\chi)v$. Thus, the game admits a threshold selling strategy, where the seller chooses to sell if her asset's value is higher than the market value of a firm that did not sell, that is, $v\geq v_2^*=h_2$.

When there are no additional future periods, the market value of a firm that did not sell is simply the expected fundamental. Thus, the threshold is defined using the following indifference condition:

$$v_2^*=h_2^*(v_2^*)=E_2[v]\left(v_2^*,q\right).$$

[15] I am indebted to an anonymous referee for this useful illustration of the model.

Notice that the set of types that sell does not directly depend on the bargaining power parameter λ or the seller's sensitivity to the market value χ.

Define $v^M(F,q)$ as the fixed point of the equality

$$v^M(F,q)=E_{F_{+1}}[v]\left(v^M(F,q),q\right)$$
$$=\frac{(1-q)E_F[v]+q\cdot F\left(v^M(F,q)\right)\cdot E_F\left[v\,|\,v\leq v^M(F,q)\right]}{1-q+q\cdot F\left(v^M(F,q)\right)}, \tag{2}$$

where F is the beginning-of-period (prior) distribution of v, and F_{+1} is the end-of-period (posterior) distribution, conditional on no disclosure.[16] The equilibrium in the static benchmark can be summarized as follows.

Lemma 1. (Last-Period Threshold). In the last period of the game, a sale occurs if and only if $v\geq v_2^*=v^M(F_1,q)$, and the market value of a firm that does not sell is $h_2^*=v_2^*$. This result is independent of bargaining power and sale prices.

A useful property of v^M, first mentioned and proven in Acharya et al. (2011) and extensively used below, is the "minimum principle."

Fact ("The Minimum Principle," Acharya et al. 2011, proposition 1)**.** For a given distribution F and probability q, $v^M(F,q)$ is unique and satisfies $v^M(F,q)=\min_{v^*} E_{F_{+1}}[v](v^*,q)$.

Given the minimum principle, the equilibrium threshold in the second period is unique for a given F_1.[17] The sale price depends, of course, both on the relative bargaining power λ and on the preference parameter χ, and is determined by Equation (1).

3. Equilibrium of the Full Model

The first step is to show that the equilibrium strategy in the first period is also a threshold strategy.

Lemma 2. (Threshold Strategy). In any possible equilibrium of the game, the equilibrium strategy in both periods is a threshold strategy.

The following proposition describes the main properties of the (threshold) equilibrium.

16 $v^M(F,q)$ is the disclosure threshold of the model in Dye (1985) and is described in Jung and Kwon (1988). The Dye model is the one-period version of the model in this paper with $\lambda=1$ and $\chi=1$. This section shows that this threshold is an equilibrium for any $\lambda\in[0,1]$ and $\chi\in(0,1]$.

17 In any threshold equilibrium, many "rejection" prices for values below the threshold ($v<v^*$) can support the equilibrium, and so the equilibrium is unique up to prices below the threshold. In what follows, I ignore this multiplicity and say that the equilibrium is unique when the set of selling types is unique.

Proposition 1. (Equilibrium Properties). When $\lambda > 0$ or $\chi < 1$, an equilibrium has the following properties:

1. Market values are decreasing:$h_0^* > h_1^* > h_2^*$.
2. Thresholds are decreasing: $v_1^* > v_2^*$.
3. Sale prices are decreasing over time: $p_1(v) > p_2(v)$ for $v > v_1^*$.
4. The average value of assets that are sold is decreasing over time: $E_0\left[v \mid v \geq v_1^*\right] > E_1\left[v \mid v \geq v_2^*\right]$.

Moreover, all the inequalities above become equalities when $\lambda = 0$ and $\chi = 1$.

In each period, no-sale is perceived by the market as a bad sign, and so it decreases the market value of the firm and thus the payoff of the seller. This, in turn, results in more pressure to sell. A seller who has an offer and chooses not to sell can expect lower market values until she sells and, in addition, a lower sale price for her asset if she chooses to sell in the future. For these reasons, the threshold is also decreasing: more seller types choose to sell as time passes. On average, better assets are sold earlier, and the expected fundamental of a sold asset decreases over time.

As one can see, the main driver of the results is that the market value of the firm decreases until a sale. This is not an immediate result, because the passage of time has two conflicting effects. On the one hand, each period the expected fundamental decreases as high-valued assets (above the threshold) are sold. On the other hand, sale prices are always below the fundamental, and so a shorter horizon and fewer opportunities to sell are actually beneficial for the firm's value. The key point of the proof is to show that, in equilibrium, the former effect is always stronger than the latter.[18]

The previous section establishes that $v_2^* = h_2^*$. I now derive explicitly the threshold in period 1. From Proposition 1 property 2 we know that a seller of types v_1^* wants to sell in period 2, and, thus, her continuation payoff if she does not sell is

$$U_1^{\mathrm{NS}}\left(v_1^*\right) = \chi h_1^* + (1-\chi)v_1^* + \beta\left[(1-q\lambda)\left(\chi h_2^* + (1-\chi)v_1^*\right) + q\lambda v_1^*\right]$$

The indifference condition $v_1^* = u_1^{\mathrm{NS}}\left(v_1^*\right) \equiv \frac{U_1^{\mathrm{NS}}(v_1^*)}{1+\beta}$ results in

$$v_1^* = \frac{h_1^* + \beta(1-q\lambda)h_2^*}{1+\beta(1-q\lambda)}. \tag{3}$$

The probability of a sale and the quality of the sold assets are independent of χ, the sensitivity of the seller's preferences to the market value (as long as $\chi > 0$). Figure 2 (a) graphically presents the derivation of the equilibrium.

[18] In the specific case in which $\lambda = 0$ and $\chi = 1$, the buyer can extract all the surplus from trade, and, thus, all seller types obtain the same equilibrium payoff in each period, whether or not they sell. The expected ownership value

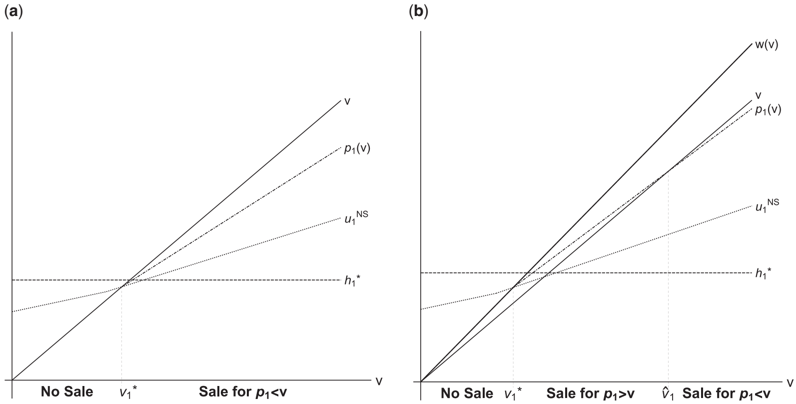

Figure 2
Graphical illustration of the equilibrium
The figure presents the first period; a similar figure could be made for the second period. Comparison of the panels shows the difference between the baseline model and the model with synergy gains. The figures are based on a numerical example with $v \sim U[0,1]$, $q=0.6$, $\chi=0.75$, $\beta=0.8$, and $\lambda=0.6$. In panel (b), $w(v)=1.2v$; $\hat{v}_1$ is defined using the equality $\hat{v}_1 = p_1(\hat{v}_1)$.

Remark 2. (Existence). The model has a finite horizon, so one can use backward induction to show that the equilibrium described above indeed exists. Section 2 shows that an equilibrium threshold exists in the last period for any prior distribution F_1. One can then turn to period 1 and, by using the last-period threshold, $v_2^* = v^M(F_1, q)$, show that v_1^* exists for any F_0. The key is to describe v_2^* as a function of F_0 and v_1^*. It is easy to extend this proof and show existence for any finite number of periods. The appendix provides a complete proof.

Remark 3. (Uniqueness). In general, the equilibrium in the model must not be unique: for a certain distribution F_0 more than one pair of thresholds $\{v_1^*, v_2^*\}$ may be an equilibrium. Multiplicity may arise because the decision of the seller today affects the beliefs of the market tomorrow and the continuation value of the seller. A high threshold today implies a better “pool“ of sellers in future periods and higher future market values, and thus a higher outside option today for the seller, which, in turn, supports the high threshold today (a similar argument can be made for a self-enforcing low threshold). The problem of multiplicity is especially pronounced when q is high, and therefore the sale threshold in period 1 has a large effect on the distribution of types in period 2.

However, additional analysis shows that uniqueness is preserved in many cases. The Online Appendix contains an analysis of the conditions for

of the firm is the same whether or not the asset is sold, and so the information that a firm did not sell in a given period does not affect market value. Thus, market value is constant over time, even though the expected fundamental decreases over time, and the sale threshold is also constant. In what follows I usually focus on the more economically meaningful case of $\lambda > 0$.

uniqueness in the two-period case. It is shown formally that, although uniqueness depends on the initial distribution of types F_0, for any distribution the equilibrium is unique if q is not too high. To show that the equilibrium is unique in a significant and large set of relevant parameters, the specific cases in which F_0 is standard normal and where it is uniform are analyzed. In the uniform case equilibrium is always unique (for any $q \in [0,1]$). In the standard normal case, I show analytically that $q < 0.57$ is a sufficient (nonbinding) condition for uniqueness, and numerically that the equilibrium is unique for any $q < 0.999$.

4. Synergy Gains

In the baseline model, value-enhancing sales cannot take place, because the seller's maximal willingness to pay is v. Sales take place because the seller cares about the market's response, that is, because of the feedback effect. The model can be easily modified to include the case in which buyers value the asset more than shareholders. This is the case, for example, if the buyers are buying a line of products that are synergic to other products that are already being sold by the firm, or if they are buying a technology that can be embedded in the firm's products (by using the term "synergy gains," I follow Bradley et al. 1983). I incorporate synergy gains by defining a function $w(v)$ that represents the value that a buyer attaches to an asset of type v, where $w(v) > v$. To remain close to the original model, I assume that $w(v)$ is identical for all potential buyers and known in advance to all market participants. Moreover, I make the following assumption on $w(v)$:

Assumption 1. The synergy gain $w(v) - v$ is (1) nondecreasing in v and satisfies (2) $\lim_{v \to \underline{V}} (w(v) - v) = 0$.

Part (1) ensures a threshold equilibrium and that v_1^* is unique for a given v_2^* and vice versa. Part (2) ensures that low types never sell, and, hence, $v_t^* > \underline{V}$ for all t.[19] An example that satisfies Assumption 1 (when $\underline{V} \in \mathbb{R}$) is $w(v) = c \cdot v - \underline{V}(c-1)$, where $c > 1$.

If synergy gains are present, the new price function is

$$p\left(v, u_t^{\mathrm{NS}}(v)\right) = (1-\lambda) \cdot u_t^{\mathrm{NS}}(v) + \lambda \cdot w(v). \tag{4}$$

Thus, the seller can obtain a price of $w(v)$ if she has all the bargaining power. Note that now a sale may take place at prices $v < p \le w(v)$, and that the

[19] A violation of part (1) may change the equilibrium: high types, if they have low synergy gains, may choose not to sell in a given period, while low types, if they have high synergy gains, may choose to sell at the same period. Such considerations are not the focus of this paper and so are assumed away. Given part (1), it is easy to prove that equilibrium has a threshold strategy: the proof follows that of Lemma (2) (with a slight change that demands that $w'(v) \ge 1$) and is omitted here for brevity. Part (2) of the assumption is made for ease of exposition: if part (2) is violated and synergy gains are high, then it is possible to have a degenerate equilibrium where all types choose to sell.

threshold types, which, by definition, are indifferent when offered the maximal price, satisfy $u_t^{\mathrm{NS}}(v_t^*) = p_t(v_t^*) = w(v_t^*)$.

It is easy to show that in this version of the model the equilibrium involves a threshold as long as Assumption 1 (1) holds. The proof is similar to the proof of Lemma 2 and thus omitted. The equilibrium of a model with "synergy gains" has the same qualitative properties as that of the baseline model, but also present some additional ones.

Proposition 2. (Synergy Gains). In a model with synergy gains as outlined above, an equilibrium satisfies all the properties of Proposition 1 (properties 1–4), as well as the following additional properties:

5. In each period, a subset of types that includes the threshold type v_t^* and types directly above it obtain a price $p_t(v) > v$.
6. There exist $\lambda_0 \in (0,1)$ and two constants $\hat{v}_1, \hat{v}_2 < \overline{V}$ such that if $\lambda < \lambda_0$, then $p_t(v) < v$ for types $v \in \left(\hat{v}_t, \overline{V}\right)$.

Property 5 is immediate from Equation (4), and merely says that, as expected, some types do sell at prices above v. The intuition behind property 6 is that when λ is below a certain level $\frac{\partial p_t}{\partial v} \in (0,1)$ and, therefore, despite property 5, high types sell at prices below v. The fact that some types sell at prices above v and some types sell at prices below v plays an important role in Section 6.1 , in which sellers choose a quantity for sale. Panel (b) of Figure 2 presents the derivation of the equilibrium. The thresholds v_1^* and $\hat{v}_1$ are marked so that one can easily see properties 5 and 6.

One can use Proposition 2 to compare the selling activity in a model with and without a feedback effect. If $\chi = 0$, that is, the seller maximizes value and do not care about the stock price, then without synergy gains there will be no trade, while with synergy gains trade will always take place, and at a price $p \in [v, w(v)]$. However, when the seller also cares about the short-term stock price ($\chi > 0$), one can see that (1) sellers with a low-value asset do not sell even if they are offered $p = w(v) > v$ and (2) sellers with a high-value asset may sell at prices $p < v$. Thus, value-destroying sales takes place even in the presence of synergy gains, and value-creating sales are forgone by some sellers.

5. Predictions and Economic Implications

5.1 Announcement returns and corporate sell-offs

In this section I focus on additional predictions of the equilibrium regarding returns. I focus on period 1 of the model, because, as is easy to show, these properties are relevant in multi-period models for any period which is not the last one.

5.1.1 Announcement returns. Propositions 1 and 2 establish that value-destroying sales take place in this model. Nevertheless, market reaction to a sale will be, on average, positive. This is established by the following result, which arises almost immediately from Proposition 1.

Corollary 1. (Positive Average Announcement Return). When a seller has some bargaining power ($\lambda > 0$), then a sale leads, on average, to a positive return, that is, $h^*_{t-1} < E_{t-1}\left[p_t(v) \mid v \geq v^*_t\right]$ for $t \in \{1,2\}$.

To see the intuition, note that the market value, which is the expected terminal value to the owner of the asset, is a martingale. In equilibrium, the market value in the beginning of period t is an average of the expected value at the end of period t if there is no sale, h^*_t, and the expected revenue from a sale in period t. Formally,

$$h^*_{t-1} = \left(1 - q + qF_{t-1}(v^*_t)\right)h^*_t + q\left(1 - F_{t-1}(v^*_t)\right) \cdot E_{t-1}\left[p_t(v) \mid v \geq v^*_t\right], \qquad (5)$$

where $q\left(1 - F_{t-1}(v^*_t)\right)$ is the probability of a sale. The corollary is immediate from 5 and the fact that $h^*_t > h^*_{t-1}$, (Proposition 1, property 1).

Corollary 1 uniquely sheds light on the possibility that, because of information asymmetries between the firm and its investors, value-destroying sales may be accompanied by a positive announcement return. In fact, value-destroying sales occur because sellers care about the announcement return. Below, I show how this may affect the interpretation of empirical results.

5.1.2 Corporate sell-offs. The results above stand in contrast to the standard "lemon market" result, in which a sale (or an attempt to sell) signals that the sold asset is of low value (a "lemon") and thus is accompanied by a negative market reaction. In general, negative price reaction for a sale arises when some traders are more informed than others (see, e.g., Kyle 1985, Glosten and Milgrom 1985).[20] The current model, therefore, better fits markets in which buyers and sellers have roughly the same information. For example, it better fits the market for corporate sell-offs, in which buyers perform a thorough process of due diligence, thus mitigating the inherent difference in information between buyers and sellers.

The empirical literature on corporate sell-offs finds that, on average, a stock response to a sale announcement is positive and tends to occur after a period of abnormally negative returns (Alexander et al. 1984; Jain 1985; Hite et al. 1987; Lang et al. 1995). Both findings are predicted by the present model.[21] Some evidence indicates that the negative performance before a sale is due to a selection bias, where firms in poor condition tend to sell some of their

[20] One example of such a market is a centralized market that comprises institutional and retail investors. Indeed, some empirical findings support a "lemon market" result in such markets (see, e.g., Kelly and Ljungqvist 2012).

[21] While the baseline model describes a sale of the entire assets of a firm, I believe it is more suitable to analyze partial sell-offs than liquidations or acquisitions. The reason is that managers often lose their job in the act of a sale, receives for a sale. This aspect is beyond the scope of this paper.

activities (Jain 1985; Lang et al. 1995). I offer an alternative information-based explanation; more empirical research is needed to try and distinguish between the two explanations.

Moreover, as discussed in the introduction, negative performance before a sale may be a result of a screening process, in which the seller first ask a high price and gradually decreases that price. Note, however, that the effect of the matching frictions, modeled by parameter q, is different in these two models. When q is high, that is, when buyers are likely to be available in the future, a screening seller has an incentive to start at a high price and decrease it slowly. In the current model, however, high q leads to lower sale thresholds, and thus lower prices. This is because when q is high, no-sale is a stronger signal that the seller is of low value. The parameter q can be interpreted as "market conditions,"that is, whether potential demand for the asset is large. Comparing the trend of returns in sales that take place when market conditions are favorable to sales that takes place when market conditions are unfavorable can thus empirically distinguish between the two theories. To the best of my knowledge, such examination is yet to be done.

All the empirical literature interprets a positive announcement return as a sign that the market believes that the sale is efficient (i.e., the asset will be better utilized by the buyer). The model above, however, generates an average positive announcement return even without any synergy gains (Corollary 1), simply because of the information externality. This calls for more caution when interpreting positive returns in asset sales. Hite et al. (1987) find that when an announced sale is unsuccessful, the seller experiences a negative abnormal return that is larger (in absolute value) than the positive bid announcement return. One interpretation of this is that the market believes failed negotiations signal low value. This is in line with the general structure of the equilibrium presented here, where sellers with low-valued assets choose not to sell.[22] Finally, from Proposition 1 I can draw a specific prediction about the pattern of returns between an announcement of an *intention* to sell and an announcement of a sale, a time window that has not been researched empirically.[23] According to the model, one would expect, from the time a firm announces its intention to sell, a flow of negative abnormal returns, net of changes due to other activities of the firm. Upon a sale announcement, one would expect an average abnormal positive return. Moreover, the distribution of sale announcement returns will be skewed to the right. This last prediction is a result of selling thresholds decreasing over time.

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[22] Hite et al. (1987) argue that cancellation should not deliver any new information. Therefore, they interpret the negative return as counterevidence for the "information hypothesis." In this paper, however, I offer a theory where both successful and unsuccessful bids release information.

[23] An example of an intention to sell is Yahoo!'s announcement on February 2, 2016, that the company would "engage on qualified strategic proposals." This was interpreted as putting its core business up for sale. See, for example, MacMillian and Mattioli (2016).

5.2 Inventories and Volume

This section extends the basic model to a case in which the seller does not sell its entire asset. I analyze sale volume within the model and show the effect of inventories.

5.2.1 A model with partial sales. Assume that a sale involves only a fraction α of the firm's asset, and $1-\alpha$ of the asset remains as inventory. Thus, the seller cares not only about the actual sale price but also about the effect that this sale has on the market value of the remaining inventory. Such an assumption fits, for example, the case in which the firm has several assets, with the same fundamental value, that cannot be sold all together. Even if the assets were not identical in value, but their values were positively correlated, the qualitative results of this section would not change.

In this section I assume α is exogenous, and endogenize it in Section 6.1. I focus on the case in which the seller has some bargaining power, that is, $\lambda > 0$. In such a case, the price is strictly increasing in v (Equation (1)) and thus discloses the value of the remaining inventory. The payoff of the seller following a sale is simply

$$\alpha \cdot p\left(v, u_t^{\text{NS}}(v)\right) + (1-\alpha)v = \alpha(1-\lambda) \cdot u_t^{\text{NS}}(v) + [1-\alpha(1-\lambda)]v. \qquad (6)$$

Thus, the equilibrium is similar to a model where the seller has more bargaining power. An inventory makes a sale more profitable, because a seller receives $p \leq v$ for the assets she sells and v for those that she retains.[24] The following lemma presents how sale thresholds are affected by higher λ and thus, also by lower α.

Lemma 3 (More Bargaining Power and/or Larger Inventory Results in Fewer Sales). In a two-period model, if the equilibrium is unique and $\lambda > 0$:

1. More bargaining power results in higher thresholds: $\frac{\partial v_t^*}{\partial \lambda} > 0$ for $t \in \{1,2\}$.
2. Selling a smaller part of the asset results in higher thresholds: $\frac{\partial v_t^*}{\partial \alpha} < 0$ for $\lambda \in (0,1)$ and $\frac{\partial v_t^*}{\partial \alpha} = 0$ for $\lambda = 1$, for $t \in \{1,2\}$.

When the seller has more bargaining power and prices are higher, or when the seller's payoff is higher due to a large retained inventory, fewer types sell and the asset takes longer to sell. The intuition is that an increase in λ or a decrease in α increase the outside option of the seller in the first period and thus leads to a higher v_1^*. Higher v_1^* results in an overall better population of types that hasn't sold in period 2, and thus a higher v_2^*.

24 It is implicitly assumed that sale prices are not affected by the presence of inventories. One might argue that a buyer who is aware of the effect of the sale on the value of the seller's inventory may use that effect in the bargaining process to pay a lower price. Results hold also for other price functions as long as one assumes that the seller's share of the surplus is increasing in the retained amount. This will be the case if one assumes that price is always positive and increasing in the value of the asset.

5.2.2 Volume and market conditions over the business cycle. The probability that the seller is matched with a buyer, q, can be interpreted as "market conditions": when conditions are good, many buyers will provide liquidity, and, thus, q is high. When market conditions are bad, liquidity dries up, and one has to wait longer for a match. The ex ante probability of a sale in each period, $q\left[1-F_t\left(v_t^*(\alpha,q)\right)\right]$, can be interpreted as volume, where $v_t^*(\alpha,q)$ is the sale threshold in period $t\in\{0,1\}$ for a given sold fraction α and probability q.[25]The size of the parameter q affects volume directly as well as indirectly through the sale thresholds v_t^*. This indirect effect also depends on the inventory of the seller $1-\alpha$. The nature of the indirect effect is described in the following lemma.

Lemma 4. (Sale Thresholds and Market Conditions). In a model with partial sales, consider two sale fractions $\alpha_l,\alpha_h\in(0,1]$ such that $\alpha_l<\alpha_h$. There exists a probability threshold $\underline{q}\in(0,1)$ such that, for any $q>\underline{q}$ and $q'<q$

1. $v_t^*(\alpha,q')>v_t^*(\alpha,q)$ and
2. $v_t^*(\alpha_l,q')-v_t^*(\alpha_l,q)>v_t^*(\alpha_h,q')-v_t^*(\alpha_h,q)$.

Lemma 4 allows us to analyze the effect of market conditions on volume. Consider first a scenario in which market conditions are very good, that is, q is close to one. In such a case no-sale is probably due to the seller's unwillingness to trade, and thus is very bad news and leads to a low market value. In equilibrium, even sellers with low-valued assets have an incentive to sell (as part of the proof of Lemma 4, it is shown that $v_t^*\to\underline{V}$ for $t\in\{1,2\}$ as $q\to 1$). Now compare that case to a market with a lower probability q': lower matching probability directly leads to less deals. Lemma 4 identifies additional *two supply-side amplification effects*. First, thresholds are higher when q is lower (Lemma 4, part 1). This further decreases volume. Second, the effect is more pronounced for a higher inventory, because higher inventory increases the sensitivity of thresholds with respect to q (Lemma 4, part 2). Because of these supply-side amplifications, even modest differences in the demand side, as evidenced in q, may result in large differences in volume.

Lemma 4 does not entail that thresholds are necessarily decreasing in q for any $q\in(0,1)$. An increase in q has two opposing effects on the outside option of the seller in the first period: first, it decreases market values in both periods, which decreases the outside option. However, it also increases the probability, and thus value, of the option to sell in the second period. The latter effect may theoretically prevail and lead to an overall increase in v_1^* (which, through its effect on the distribution in the second period, may also increase v_2^*).

[25] More accurately, the probability of a sale in each period creates a distribution on the ex ante expected time of a sale, which can be interpreted as volume. If both selling thresholds increase, then the new distribution is dominated by the original distribution in the first-order sense.

Appendix A.7 shows that the thresholds are decreasing in q for any $q \in (0,1)$ when λ and/or β are small enough (and thus the value of the outside option is low enough). For some distributions (e.g., uniform), there is no need to limit parameters.

Moreover, despite the fact that the model in this paper assumes a constant and known q, the model can be extended to include a time-varying q. Appendix A.7.3. outlines a model where q fluctuates randomly over time between a high and a low value. I show that, as long as there is some persistence in market conditions (i.e., $\Pr(q_2 = q_1) \geq 0.5$), a result similar to Lemma 4 part 1 can be proven. Thus, negative demand shocks are amplified even in a model where market conditions may recover in the future. This prediction may be used to explain why the decrease in demand for structured assets during 2007, amplified by large inventories of such assets held by financial institutions, has resulted in a sharp contraction in volume and "market dry-up" during the 2008 financial crisis.

Milbradt (2012) and Bond and Leitner (2015) show, as this paper, that inventories may prevent asset sales in over-the-counter markets. Both papers deal with firms that have asset-backed loans. These firms forgo profitable transactions when such transactions decrease the value of their inventory, to avoid a violation of their capital constraint. By contrast, the result here does not rely on the existence of leverage, but rather demands a certain negative demand shock to generate the dry-up. Moreover, a cross-sectional examination of the "market dry-up" in structured asset market during the 2008 financial crisis could potentially distinguish between the theories: this paper suggests that even banks that were not leverage constrained were reluctant to sell. To the best of my knowledge, such an examination has yet to be performed.

Milbradt (2012) presents a model without asymmetric information in which the value of the inventory is updated due to fair value accounting of illiquid ("level 3") assets and does not change when there are no transactions. Bond and Leitner (2015) is closer to the present paper, as it analyzes a standard "lemon market" in which the uninformed buyer is leveraged and has an inventory. Moreover, in Bond and Leitner (2015) the occurrence of a market dry-up is related to the inventory's *absolute* value and thus, for example, an increase in the average quality of assets will rejuvenate the market. By contrast, in this paper what matters is the relation between the *expected* value of the firm's assets and their actual value, and, therefore, an improvement of the pool will result in higher thresholds and will not necessarily increase trading volume.

6. Additional Extensions

6.1 Endogenous retention

The analysis of a model with partial selling (Section 5.2) has shown that lower α is better for the seller, as it is equivalent to more bargaining power. Thus, if the seller could, she would choose the minimal possible α (this will be true whether

α is chosen at the beginning of the game or in the beginning of each period). Notice the difference between the present model and previous ones, such as Leland and Pyle (1977) and Demarzo and Duffie (1999), in which sellers signal their high value by retaining part of the asset. In contrast to these models, here such signaling is not possible because all types prefer to retain as much of the asset as possible and sell a minimal amount. In other words, here sale prices are not affected by the retention decision because the buyer is informed.[26]

However, this result does not hold when one considers the possibility of "synergy gains," as described in Section 4. In a model with synergy gains, some types obtain a price $p_t(v) > v$, and may thus have an incentive to maximize the share they sell. I now explore this formally.

Consider a model similar to the one in Section 5.2, except that every time the seller meets a buyer she chooses between two possible fractions $0 < \alpha_L < \alpha_H \leq 1$ (assuming two fractions is without loss of generality, as intermediate fractions will not be chosen in equilibrium). The following Lemma summarizes the equilibrium in this model when the buyer's bargaining power is strong enough ($\lambda < \lambda_0$, where λ_0 is defined in the appendix).

Lemma 5. (Endogenous Retention). In a model with synergy gains and endogenous retention, when $\lambda \in (0, \lambda_0)$, then in each period $t \in \{1,2\}$ there exists a type $\hat{v}_t < \overline{V}$, defined using the equality $\hat{v}_t = p_t\left(\hat{v}_t\right)$, such that:

1. A seller with an asset of value $v < v_t^*$ does not sell;
2. A seller with an asset of value $v \in [v_t^*, \hat{v}_t)$ sells the maximal amount α_H if a buyer arrives; and
3. A seller with an asset of value $v \geq \hat{v}_t$ sells the minimal amount α_L if a buyer arrives.

Figure 3 illustrates the nature of the equilibrium. Assets of value $v \in [v_t^*, \hat{v}_t)$ are sold for prices $p_t(v) > v$, and, thus, the seller wants to sell as much as possible. In contrast, assets of value $v \geq \hat{v}_t$ are sold for $p_t(v) < v$; for those assets, the seller sells only to reveal her asset's value to the market and thus is interested in selling the minimal amount. Note that $\hat{v}_t < \overline{V}$ only when the buyer has enough bargaining power.[27]

For an observed sale at a given time t, the model predicts a negative correlation between the size of the deal and the value of the asset: assets of

[26] Because retention is profitable in this model, one might ask whether it is possible to have a separating equilibrium, where *low* retention is used to signal higher value. Though low retention is costly, such an equilibrium is not feasible because the game does not display the "single-crossing property": low retention is more costly to higher types than to lower types because higher types sell earlier.

[27] The current analysis assumes that the retained inventory is utilized by the seller. A possible extension is to allow a sale of the remaining inventory to another buyer at the end of the game. In that case, the payoff of the seller from her leftover inventory is $\lambda w(v)+(1-\lambda)v$ rather than v. This does not change the qualitative nature of the results: the threshold type v_t^* sells at the price $p_t(v_t^*)=w(v_t^*)$, meaning that this type, as well as the set of types just above it, prefer to sell a high fraction α_H of the asset.

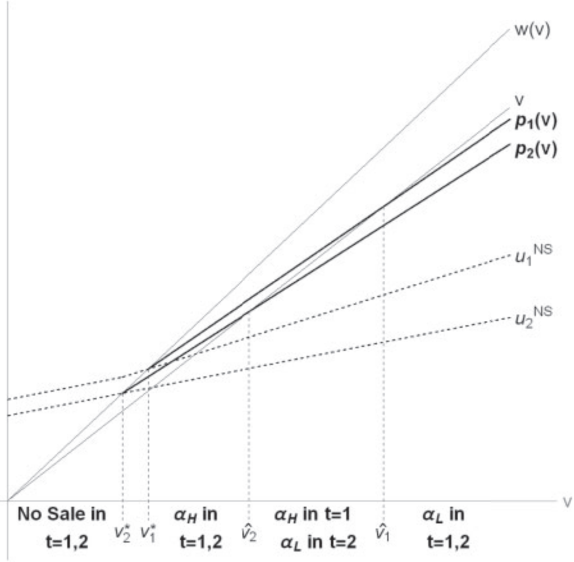

Figure 3
Retention with synergy gains
v_t^* represents the sale threshold in period t, and $\hat{v}_t$ is defined using the equality $\hat{v}_t = p_t\left(\hat{v}_t\right)$. The figure was generated from the same numerical example used in Figure 2. Because the seller obtains p_t on the sold quantity (α^j) and v on the retained quantity, types $v \geq \hat{v}_t$ prefer to retain more. The quantity that maximizes the seller's payoff in each interval (α^L or α^H) is specified below the horizontal axis. A seller of type $v \in \left[v_2^*, v_1^*\right]$ does not sell in period 1 and sells α^H in period 2.

higher value will be associated with a smaller deal and higher leftover inventory. This cross-section prediction is similar to the predictions of Leland and Pyle (1977) and Demarzo and Duffie (1999), but, as mentioned above, this prediction is not due to a signaling effect. Instead, it relies on the synergy gains as well as the bargaining power of the informed buyer. Synergy gains allow some types to sell the asset at a price $p > v$, and those types prefer to sell as much of the asset as possible. The buyer's bargaining power leads other types to sell at $p < v$, and those types sell the minimal amount.

Because $p_2(v) < p_1(v)$ (by part (3) of Proposition 1), it follows that $\hat{v}_2 < \hat{v}_1$; that is, the model predicts that, for some types, retention decreases over time. Figure 3 illustrates this. A model with many periods (such as the one described in the previous version of this paper) predicts that the game is divided into three stages, each comprising several periods, that differ in terms of the retention decision. In the first stage the seller refuses to sell. In the second stage, the seller sells the maximal fraction if she is matched with a buyer. However, if a buyer arrives only in the third stage of the game, the seller will sell the minimal fraction.

6.2 Buyers care about market value

This section analyzes a model where the buyer's payoff, like the seller's, depends on the beliefs of the market about the quality of the asset. This could happen, for example, if the buyer, like the seller, is a public firm, and its manager and/or shareholders care about its market value. Formally, assume that the buyer obtains a payoff of $\chi \cdot E\left[v \,|\, \text{Sale for price } p_t\right] + (1-\chi)v - p_t$ if he buys the asset, and zero otherwise. $E\left[v \,|\, \text{Sale for price } p_t\right]$ is the "market valuation" of the asset following a sale. For symmetry I assume that both seller and buyer put the same weight, χ, on the beliefs of the market, but the analysis can be easily extended to different weights.

In the baseline model, the surplus from trade is $v - u_t^{\text{NS}}$, and the price p only determines the division of this surplus according to the relative bargaining power. The price is therefore strictly increasing in v, and a sale discloses the value of the asset v. In this extension, however, the surplus from trade may depend on the price, as it is $\chi \cdot E\left[v \,|\, \text{Sale for price } p_t\right] + (1-\chi)v - u_t^{\text{NS}}$. The trading parties may have a joint incentive to choose a price that will increase the market evaluation $E\left[v \,|\, \text{Sale for price } p_t\right]$. The price now may only *signal* value, and the quality of this signal is part of the equilibrium.

Because the price determines both the surplus and the division of the surplus, the simple price function in Equation (1) can no longer represent the bargaining process well. This is because it is possible that one of the parties will accept a price that gives him a lower share of the surplus if that price signals a higher value to the market (and thus results in a higher surplus). One needs to specify explicitly how prices are chosen by the trading parties, and specifying different bargaining protocols may lead to different equilibria. For concreteness, and to avoid bargaining issues and focus on the role of the buyer's preferences, *this section assumes that* $\lambda = 1$. That is, the buyer simply accepts or rejects the price p that is asked by the seller. Upon making his decision, the buyer takes into account the market valuation following a sale with a price p.

6.2.1 Original equilibrium survives. The first result is that the equilibrium of the baseline model is still an equilibrium of the extended model.

Lemma 6. In a model where the buyer's compensation is also sensitive to market valuation and the seller has full bargaining power, there is an equilibrium where $p_t(v) = v$. Moreover, this is the only equilibrium where $p_t(v)$ is strictly monotone for all t.

The lemma above shows that in the "trade game" between buyer and seller, the unique separating equilibrium is to same as the equilibrium of the baseline model with $\lambda = 1$. The intuition behind the proof of Lemma 6 is that in the case in which the market believes that $p = v$, a price of v is the maximum that a seller of type v can ask for. No other monotone price function can be an equilibrium as some seller types can profit by deviating to $p = v$.

6.2.2 Additional equilibria have unreasonable out-of-equilibrium beliefs. Additional pooling or semipooling equilibria, that is, equilibria where some assets of different qualities are sold for one single price p, are, however, possible. One can establish that, for all possible equilibria, prices of sold assets are weakly increasing in value.

Lemma 7. In any equilibrium, for any two types v and v' that choose to sell in period t if a buyer arrives, $v < v'$ iff $p_t(v) \leq p_t(v')$.

Proof. Consider an equilibrium where $p_t(v) = p$ and denote by $\mathcal{V}$ the set of types that sell at a price p. Now consider the type $v' > v$. Note that

$$p \leq \chi E[v \mid v \in \mathcal{V}] + (1-\chi)v < \chi E[v \mid v \in \mathcal{V}] + (1-\chi)v';$$

therefore, if type v' asks for p, the buyer will agree. Thus, in equilibrium it must be that $p_t(v') \geq p$. ■

Consider an equilibrium with a set $\mathcal{V}$ such that sellers of type $v \in \mathcal{V}$ sell in period t if a buyer arrives and $p_t(v) = p_t(v') = p_{\mathcal{V}}$ for all $v, v' \in \mathcal{V}$. A corollary of Lemma 7 is that all prices $p \in (p_{\mathcal{V}}, v_h]$, where $v_h = \max_v \{v \in \mathcal{V}\}$, are not used in this equilibrium.[28] An equilibrium with pooling can only supported by an out-of-equilibrium belief that any set of types who sell at a price $p \in (p_{\mathcal{V}}, v_h)$ has an expected value that is lower than $E[v \mid \mathcal{V}]$. Else, types in $\mathcal{V}$ would not find it optimal to ask for $p_{\mathcal{V}}$. In other words, all equilibria that involve pooling must contain nonmonotone beliefs: the market does not believe that a higher price necessarily signals a higher asset's value (although this is always the case on the equilibrium path). Because nonmonotone beliefs are less reasonable in the context of this game, these pooling equilibria seems to have less economic meaning.

6.2.3 A refinement. Because the trading stage is not a standard signaling game – there is no actual "cost" of signaling, and the price is limited by the buyers' consent – the pooling of types to a single price cannot be eliminated by a standard refinement, such as "divinity" or the D1 criterion. I offer, instead, a simple and natural condition on equilibrium that eliminates all equilibria but the one described in Lemma 6.

I perturb the game and assume that a small mass $\varepsilon \to 0$ of the potential buyers are *insensitive* to the market beliefs and obtain a payoff of v from buying an asset of quality v (i.e., they are similar to the buyers in the baseline model). Moreover, the "type" of the buyer (sensitive or insensitive) is observed by the seller before making the offer and is not observed by the market. In this perturbed game, there may be pooling when the seller faces "sensitive" buyers, but it is always optimal for the seller to charge "insensitive" buyers $p = v$. As

[28] According to Lemma 7, prices above p' may only be asked in equilibrium by seller types that are greater than v_h. However, had any of those prices been used in equilibrium, it wouldn't have been optimal for type v_h to ask for $p_{\mathcal{V}}$.

a result, if the market observes a price p that never occurs when selling to "sensitive" buyers, then the market believes the sold asset is of type $v=p$. Formally, such perturbation imposes the following additional condition on the out-of-equilibrium beliefs of the original game:

Condition (P0). Let $\mathcal{P}_t$ be the set of all sale prices in equilibrium at period t. For every price $p\in(\underline{V},\overline{V})$ but $p\notin\mathcal{P}_t$, the market belief after observing p is $v=p$.

If one imposes condition (P0), then no pooling or semipooling equilibrium is possible. To understand why, note that if a set of types $v\in\mathcal{V}$ pool and sell at a price $p_{\mathcal{V}}$ in period t, then $p_{\mathcal{V}}\leq\chi\cdot E[v\,|\,v\in\mathcal{V}]+(1-\chi)v_l$, where $v_l=\min_v\{v\in\mathcal{V}\}$. From Lemma 7 it is clear that all types $v\in(p_{\mathcal{V}},v_h)$ have an incentive to ask for a price $p=v$ rather than $p_{\mathcal{V}}$.[29]

6.3 Compensation with a vesting period

One possible interpretation of the seller's payoff function, as discussed in Section 1, is that it represents the preferences of a delegated manager that cares about short-term price effects, and is in conflict with shareholders that only care about (long-term) value. Under such interpretation, value-destroying sales only occur because of the agency problem between the manager and the shareholders, and Section 1 discusses why stock-sensitive compensation may, nevertheless, arise. Following the 2008 financial crisis, public concern rose regarding the effect of unrestricted short-term performance-based compensation. Some investors and policy makers have argued for the adoption of executive compensation schemes with long vesting periods to ensure that managers will only care about the medium and long runs and prevent manipulation of short-term performance. Several academic papers support this suggestion (see, e.g., Bhagat and Romano 2009, Edmans et al. 2012, Peng and Röell 2014).

In light of this suggestion, it is natural to ask whether a vesting period can also mitigate the conflict of interest between the manager and the shareholders in this paper. The answer to this question is no. In any case in which the seller's compensation is related to the market value, even if this relation does not start in period 1, the model predicts value-destroying sales in *all* periods.[30]

To highlight this point, I analyze a model with a vesting of one period. That is, the seller has some $\chi>0$ only in the last period, and in the first period it is

[29] The refinement offered here is similar to the "truth-leaning equilibrium" offered by Hart et al. (2017). Their definition includes, in addition to condition (P0), an additional condition, (A0), that is irrelevant in the present setup. Hart et al. show that this refinement is used, sometimes implicitly, in many papers that deal with disclosure games.

[30] Of course, a vesting of all periods, equivalent to $\chi=0$, prevents any sales. This section, like other papers, focuses on the case in which there are reasons to keep the manager's compensation sensitive to the stock-price in the medium and long run. See also the discussion on seller's preferences in Section 1.

assumed that $\chi = 0$. Thus, the manager cares about h_2 but not on h_1. I obtain the following result.

Lemma 8. In a model with a vesting period, (1) there is a threshold equilibrium in both periods, (2) $v_1^* = v_2^*$, and (3) thresholds are chosen such that the probability of a sale is maximized.

Lemma 8 shows that in a model with a vesting period, value-destroying sales take place also in the periods where the manager's compensation is not vested. One can show that in a model with more than two periods, thresholds are constant until the first period after the vesting period, and are decreasing after that. Intuitively, the subgame after the vesting period is similar to the baseline model. Thus, during the vesting period the quality of sold assets is not decreasing. Moreover, part (3) of the lemma entails that a vesting of one period actually maximizes the volume of value-destroying sales. Thus, if shareholders attempt to prevent value-destroying sales, one-period vesting is the worst thing they can do.

7. Concluding Remarks

In this paper I have analyzed a dynamic model of over-the-counter asset sales. The focus of the model is the information that is revealed through a sale when buyers are informed, and how market's reaction to this information feeds back to the seller's decision to sell, when her compensation is sensitive to the stock price. The focus on the dynamics and effect of information has allowed me to draw some specific predictions regarding the pattern of returns, which I believe are able to shed light on several markets, such as markets for corporate sell-offs and markets for structured financial assets. To generate sharp predictions, I have assumed away many aspects of over-the-counter markets: the possibility of a competition between sellers and buyers, search frictions, the role of market makers, etc. A challenge left for future research is to develop a more general model of over-the-counter markets where traders care not only about their gains from trade but also about the information revealed as part of this trade.

Appendix

A.1 Preliminaries

It is useful to explicitly present the updated belief of the market following no-sale in a threshold equilibrium. The partial and cumulative distribution functions at the end of period $t \in \{1,2\}$, following no-sale, are

$$f_t\left(v; v_t^*, q, f_{t-1}\right) = \begin{cases} \frac{f_{t-1}(v)}{1-q+qF_{t-1}(v_t^*)} & v < v_t^* \\ \frac{(1-q)f_{t-1}(v)}{1-q+qF_{t-1}(v_t^*)} & v \geq v_t^* \end{cases} \quad \text{and}$$

$$F_t\left(v;v_t^*,q,F_{t-1}\right)=\begin{cases}\frac{F_{t-1}(v)}{1-q+qF_{t-1}(v_t^*)} & v<v_t^*\\ \frac{qF_{t-1}(v_t^*)+(1-q)F_{t-1}(v)}{1-q+qF_{t-1}(v_t^*)} & v\geq v_t^*.\end{cases} \tag{A.1}$$

In a threshold equilibrium, f_t and F_t can be written as a functions of past thresholds $\left\{v_\tau^*\right\}_{\tau=t}^2$ and the prior F_0. For brevity, in the sequel, I omit all variables, except v_t^*, when the context is clear.

A.2 Proof of Lemma 2

Proof. Section 2 shows that the property holds for the second period. For the first period, assume an arbitrary strategy $V_1=\{v\,|\,v$ sells in period 1$\}$, and denote by $h_1(V_1)$ the resultant market value of a firm that hasn't sold until the end of period 1. A seller of type v' sells if and only if $v'\geq u_1^{\mathrm{NS}}(v')$. The expected payoff of the seller if she does not sell is

$$U_1^{\mathrm{NS}}(v)=\chi h_1(V_1)+(1-\chi)v+\beta\left[(1-q)u_2^{\mathrm{NS}}(v)+q\max\left\{u_2^{\mathrm{NS}}(v),p_2(v)\right\}\right].$$

Substituting (1), the outside option $u_1^{\mathrm{NS}}(v')$ is

$$u_1^{\mathrm{NS}}(v)=\frac{1}{1+\beta}\left[\chi h_1(V_1)+(1-\chi)v\right]+\frac{\beta}{1+\beta}\left[(1-q\lambda)\cdot u_2^{\mathrm{NS}}(v)+q\lambda\max\left\{u_2^{\mathrm{NS}}(v),v\right\}\right].$$

Given that $\frac{\partial u_2^{\mathrm{NS}}(v)}{\partial v}=1-\chi$, then $\frac{\partial u_1^{\mathrm{NS}}(v)}{\partial v}\in(0,1)$ (except for $v=v_2^*$, where the function is continuous, but not differentiable). Thus, $\frac{\partial u_1^{\mathrm{NS}}(v)}{\partial v}\leq\frac{\partial p_1(v)}{\partial v}=(1-\lambda)\frac{\partial u_1^{\mathrm{NS}}(v')}{\partial v'}+\lambda$, and if type v' prefers to sell so do all types $v>v'$. ■

A.3 Proof of Proposition 1

Proof. I prove the properties for the case in which the buyer valuation of an asset of type v is $w(v)\geq v$, and $w(v)$ satisfies Assumption 1. Thus, I prove that properties 1-4 hold in the basic model where $w(v)=v$ (Proposition 1) and in a model with synergy gains (first four properties in Proposition 2).

The proof contains a series of Lemmas (I prove for $\lambda>0$, it is easy to show that all the inequalities become equalities when $\lambda=0$).

Lemma 9. (Property 1 (a)). $h_1^*>h_2^*$.

Proof. Substitute (4) and $u_2^{\mathrm{NS}}(v)=\chi h_2^*+(1-\chi)v$ into (5) to obtain

$$\begin{aligned}h_1^*=&\left[1-q\left(1-F_1(v_2^*)\right)(1-\chi(1-\lambda))\right]h_2^*\\&+q\left(1-F_1(v_2^*)\right)(1-\chi(1-\lambda))\cdot E_1\left[\frac{(1-\lambda)(1-\chi)}{1-\chi(1-\lambda)}v+\frac{\lambda}{1-\chi(1-\lambda)}\cdot w(v)\,|\,v\geq v_2^*\right].\end{aligned} \tag{A.2}$$

By construction,

$$\begin{aligned}E_1[v]&=\left(1-q+qF_1(v_2^*)\right)E_1[v\,|\,\mathrm{NS}]+q\left(1-F_1(v_2^*)\right)\cdot E_1[v\,|\,\mathrm{Sale}]\\&=\left(1-q+qF_1(v_2^*)\right)h_2^*+q\left(1-F_1(v_2^*)\right)\cdot E_1\left[v\,|\,v\geq v_2^*\right],\end{aligned}$$

and so $h_2^*<E_1[v]<E_1\left[v\,|\,v\geq v_2^*\right]$. Because $w(v)\geq v$, then $h_2^*<E_1\left[\alpha v+(1-\alpha)\cdot w(v)\,|\,v\geq v_2^*\right]$ for any $\alpha\in[0,1]$. This fact, together with (A.2), entails the desired result. ■

Lemma 10. (Property 2). $v_1^* > v_2^*$.

Proof. First, note that in the last period, when there may be synergy gains, the threshold is determined using the indifference condition

$$w(v_2^*) = u_2^{NS}(v_2^*) = \chi h_2^* + (1-\chi)v_2^*.$$

Now assume that, in contrary to the lemma, $v_1^* \leq v_2^*$. Then type v_2^* weakly prefers to sell in period 1, that is,

$$\chi h_1^* + (1-\chi)v_2^* + \beta \cdot w(v_2^*) \leq (1+\beta) \cdot p_1\left(v_2^*\right).$$

Substituting h_2^* for h_1^*, and given Lemma 9, then

$$(1+\beta) \cdot w(v_2^*) = \chi h_2^* + (1-\chi)v_2^* + \beta \cdot w(v_2^*) < (1+\beta) \cdot p_1\left(v_2^*\right),$$

or $w(v_2^*) < p_1\left(v_2^*\right)$, a contradiction to (4). ■

Lemma 11. (Property 3). For types $v > v_1^*$, $p_1(v) > p_2(v)$.

Proof. By definition (Equation (4)), $p_1(v) > p_2(v)$ iff $u_1^{NS}(v) > u_2^{NS}(v)$. I now show this is indeed the case for types $v > v_2^*$ (and thus, given Lemma 10, for all types $v > v_1^*$).

From Lemma 10 we know that $u_2^{NS}(v_2^*) = w(v_2^*) < u_1^{NS}(v_2^*)$. Types above v_2^* want to sell in the second period, and, thus, their outside option in the first period is

$$\begin{aligned} u_1^{NS}(v)|_{v \geq v_2^*} &= \frac{1}{1+\beta}\left[\chi h_1^* + (1-\chi)v + \beta\left((1-q)u_2^{NS}(v) + q \cdot p_2(v)\right)\right] \\ &= \frac{1}{1+\beta}\left[\chi h_1^* + (1-\chi)v + \beta\left((1-q\lambda)\left(\chi h_2^* + (1-\chi)v\right) + q\lambda w(v)\right)\right] \\ &= \chi\frac{h_1^* + \beta h_2^*}{1+\beta} + (1-\chi)v + \frac{\beta}{1+\beta}q\lambda\left(w(v) - \chi h_2^* - (1-\chi)v\right). \end{aligned} \tag{A.3}$$

Observe that

$$\frac{\partial u_1^{NS}(v)}{\partial v}|_{v > v_2^*} = \frac{1+\beta(1-q\lambda)}{1+\beta}(1-\chi) + \frac{\beta q\lambda}{1+\beta}w'(v). \tag{A.4}$$

Because $w'(v) > 1-\chi$ (Assumption 1 (1)), then

$$\frac{\partial u_1^{NS}(v)}{\partial v}|_{v > v_2^*} > 1-\chi = \frac{\partial u_2^{NS}(v)}{\partial v}.$$

Thus, $u_1^{NS}(v) > u_2^{NS}(v)$ also for $v > v_2^*$. ■

Lemma 12. (Property 1 (b)). $h_0^* > h_1^*$.

Proof. From Lemma 9 and (5) we obtain

$$h_2^* < h_1^* < E_1\left[p_2(v) \mid v \geq v_2^*\right].$$

From Lemma 10

$$E_1\left[p_2(v) \mid v \geq v_2^*\right] < E_1\left[p_2(v) \mid v \geq v_1^*\right],$$

and from Lemma 11

$$E_1\left[p_2(v)\mid v\geq v_1^*\right]<E_1\left[p_1(v)\mid v\geq v_1^*\right].$$

Finally, because in the first period a fixed fraction q of types above v_1^* sell, then the density function conditional on $v>v_1^*$ stays the same, that is, $\frac{f_0(v)}{1-F_0(v_1^*)}=\frac{f_1(v)}{1-F_1(v_1^*)}$. Thus

$$E_1\left[p_1(v)\mid v\geq v_1^*\right]=E_0\left[p_1(v)\mid v\geq v_1^*\right].$$

This list of inequalities entail that

$$h_1^*<E_0\left[p_1(v)\mid v\geq v_1^*\right].$$

Equation (5) for $t=1$ is

$$h_0^*=\left(1-q+qF_0(v_1^*)\right)h_1^*+q\left(1-F_0(v_1^*)\right)\cdot E_0\left[p_1(v)\mid v\geq v_1^*\right],$$

which entails $h_0^*>h_1^*$. ■

Note that Lemmas 9 and 12 together form property 1.

Lemma 13. (Property 4). $E_0\left[v\mid v\geq v_1^*\right]>E_1\left[v\mid v\geq v_2^*\right]$.

Proof. The same argument made in the proof of Lemma 12 entails

$$E_0\left[v\mid v\geq v_1^*\right]=E_1\left[v\mid v\geq v_1^*\right].$$

Given Lemma 10,

$$E_1\left[v\mid v\geq v_1^*\right]>E_1\left[v\mid v\geq v_2^*\right],$$

which leads to the desired result. ■

A.4 Proof of Existence of Equilibrium

Lemma 2 and Proposition 1 describe the properties that any possible equilibrium must have. I now show such an equilibrium exists. A version of the proposition below for a general number of periods appears in a previous working paper version.

Proposition 3. A threshold equilibrium with the properties described in Proposition 1 exists.

Proof. I show that for any prior F_0 one can find a sequence of thresholds $\left\{v_\tau^*\right\}_{\tau=1}^2$ that satisfy (3), where market values (for a firm that hasn't sold) $\left\{h_\tau^*\right\}_{\tau=0}^2$ satisfy (5).

1. Consider the one-period subgame that starts after a first period with a threshold v_1, where the initial distribution of types is F_0. Denote by $\hat{v_2}(F_0,v_1,q)$ and $\hat{h}_2(F_0,v_1,q)$ the equilibrium threshold and market value in that subgame. For a given q, F_0 and v_1 determine the distribution of types in the beginning of the subgame, $F_1(F_0,v_1,q)$, according to (A.1). From the analysis in Section 2 we know that $\hat{v_2}(F_0,v_1,q)=\hat{h}_2(F_0,v_1,q)=v^M(F_1(F_0,v_1),q)$.
2. Denote by $\hat{h}_1(F_0,v_1,\lambda,q)$ the market value of a firm who hasn't sold at the first period as a function of the game history and parameters. Using (A.2) and the functions above, we can write h_1 as a function of v_1 and the parameters:

$$\hat{h}_1(F_0,v_1,\lambda,q)=\left[1-q\left(1-F_1(\hat{v_2}(F_0,v_1),q)\right)(1-\chi(1-\lambda))\right]\hat{h}_2(F_0,v_1,q)$$
$$+q\left(1-F_1(\hat{v_2}(F_0,v_1,q),q)\right)(1-\chi(1-\lambda))\cdot E_1\left[v\mid v\geq\hat{v_2}(F_0,v_1,q)\right]. \quad \text{(A.5)}$$

3. Finally, the threshold v_1^* satisfies Equation (3), that is
$$v_1^*(F_0,\lambda,q)=\frac{\hat{h}_1\left(F_0,v_1^*(F_0,\lambda),\lambda,q\right)+\beta(1-q\lambda)\hat{h}_2\left(F_0,v_1^*(F_0,\lambda),q\right)}{1+\beta(1-q\lambda)}. \tag{A.6}$$
To see that a fixed point solution v_1^* exists for any initial distribution F_0 note that (1) in a threshold equilibrium $h_\tau \le E_\tau[v] < E_0[v]$ for $\tau \in \{1,2\}$; (2) moreover, $h_\tau > \underline{V}$ (to see this, observe that there is always a positive probability that a buyer hasn't arrived until period τ). Thus, the RHS of (A.6) is bounded by the interval $\left(\underline{V}, E_0[v]\right)$, and due to continuity, there exists a value $v_1^* \in \left(\underline{V}, E_0[v]\right)$ that satisfies (A.6). The second period threshold satisfies $v_2^*=\hat{v}_2(F_0,v_1^*)$. This concludes the proof. ■

A.5 Proof of Proposition 2

Proof. The first four properties are already proven for the case of synergy gains in the proof of Proposition 1 (Appendix A.3). I now prove properties 5 and 6. Note that below I use the fact that $w(v)>v$, and thus these properties are relevant only to a model with synergy gains.

Property 5 is immediate from the fact that in each period the threshold type satisfies $p_t(v_t^*)=w(v_t^*)>v_t^*$. This is due to the efficiency of equilibrium. Because of the continuity of the price function, this is true also for some types who are greater than v_t^*.

To prove property 6, differentiate the price function (4) with respect to v to obtain
$$\frac{\partial p_t}{\partial v}=(1-\lambda)\cdot\frac{\partial u_t^{\text{NS}}}{\partial v}+\lambda\cdot w'(v).$$
Note that (1) because $u_2^{\text{NS}}=\chi h_2+(1-\chi)v$ then $\frac{\partial u_2^{\text{NS}}}{\partial v}=1-\chi$; (2) By Equation (A.4), $\frac{\partial u_1^{\text{NS}}}{\partial v}\in\left[1-\chi,w'(v)\right)$, depending on λ, and $\frac{\partial^2 u_1^{\text{NS}}}{\partial\lambda\partial v}>0$. Therefore, $\frac{\partial p_t}{\partial v}\mid_{\lambda=0}=1-\chi$ and $\frac{\partial^2 p_t}{\partial\lambda\partial v}>0$ for $t=\{1,2\}$. Thus, there exists a series of parameters $\lambda_t^1\in(0,1)$, $t\in\{1,2\}$, such that $\frac{\partial p_t}{\partial v}<1$ for $\lambda<\lambda_t^1$. If $\overline{V}=\infty$, then we can set $\lambda_0=\min\{\lambda_t^1\}_{t=1,2}$ to obtain property 6. For $\overline{V}<\infty$ define $\lambda_t^{\overline{V}}<\lambda_t^1$ as the value that satisfies
$$(1-\lambda_t^{\overline{V}})u_t^{\text{NS}}(\overline{V};\lambda_t^{\overline{V}})+\lambda_t^{\overline{V}}\cdot w\left(\overline{V}\right)=\overline{V}.$$
To see that $\lambda_t^{\overline{V}}>0$, it is sufficient to show that when $\lambda=0$, $p_t(\overline{V})=u_t^{\text{NS}}(\overline{V})<\overline{V}$, which is immediate. Thus, $\lambda_t^{\overline{V}}\in(0,1)$ for $t=1,2$. Finally, we obtain property 6 by defining $\lambda_0=\min\{\lambda_t^{\overline{V}}\}_{t=1,2}$. ■

A.6 Proof of Lemma 3

Proof. I first prove part one of the lemma using two steps. First I prove that $\frac{\partial v_1^*}{\partial\lambda}>0$. From Lemma 1 we know that v_2^* does not directly depend on λ, and so λ affects it only through the distribution F_1, that is, through v_1^*. Thus, in step 2, I prove that $\frac{\partial v_2^*}{\partial v_1}\mid_{v_1=v_1^*}>0$, which, together with step 1, entails that $\frac{\partial v_2^*}{\partial\lambda}>0$.

Step 1: $\frac{\partial v_1^*}{\partial\lambda}>0$. Consider the fixed point solution $v_1^*(\lambda)$ to (A.6). Throughout the proof I assume q and F_0 are given and thus omit this variables. Now suppose λ increase by a small amount $\epsilon>0$ to $\lambda+\epsilon$. To prove that $v_1^*(\lambda)<v_1^*(\lambda+\epsilon)$ it is sufficient to show that the RHS of (A.6) is increasing in λ. Note $\hat{h}_2(F_0,v_1)$ is independent of λ (Lemma 1). Thus, we can decompose the total effect on the RHS to two: (1) for given market values, the weight of $\hat{h}_1\left(v_1^*,\lambda\right)$ increases and the weight of $\hat{h}_2\left(v_1^*\right)$ decreases; (2) $\hat{h}_1(F_0,v_1^*,\lambda)$ changes to $\hat{h}_1(F_0,v_1^*,\lambda+\epsilon)$. Effect (1) increases the RHS because $\hat{h}_1(F_0,v_1^*,\lambda)>\hat{h}_2(F_0,v_1^*)$ (Proposition (1) property 1). To see that $\frac{\partial\hat{h}_1}{\partial\lambda}>0$ and thus effect (2) also increases the RHS consider Equation (A.5). An increase in λ decreases the weight of $\hat{h}_2$ and increases the weight of $E_1\left[v\mid v\ge\hat{v}_2(F_0,v_1)\right]$. This clearly increases $\hat{h}_1$ (Proposition (1) property 1). Thus, I conclude that the RHS is increasing in λ and thus also v_1^*.

Step 2: $\frac{\partial v_2^*}{\partial v_1}|_{v_1=v_1^*} > 0$.

1. There exists a value $\underline{v} \in (\underline{V}, \overline{V})$ such that $\underline{v} = \hat{h}_2(\underline{v})$. This is because $\lim_{v_1 \to \underline{V}^+} F_1(v; v_1, q) = \lim_{v_1 \to \overline{V}^-} F_1(v; v_1, q) = F_0(v)$, and thus $\lim_{v_1 \to \overline{V}^-} \hat{h}_2(v_1) = \lim_{v_1 \to \underline{V}^+} \hat{h}_2(v_1) = v^M(F_0, q)$. By continuity, $\underline{v} \in (\underline{V}, \overline{V})$.
2. $\hat{h}_2(v_1)$ attains a (global) minimum in $\underline{v}$:
   - (a) Denote by $E_2[v](v_2; v_1)$ the expected fundamental at the end of period 2 given that the thresholds in both periods are v_1 and v_2. From Equation (2) we can write $\hat{h}_2(v_1) = E_2[v]\left(\hat{h}_2(v_1); v_1\right)$, and so $\underline{v} = E_2[v]\left(\underline{v}; \underline{v}\right)$.
   - (b) Suppose that the sale threshold in period 1 is lower, $v_1 = v' < \underline{v}$, but that $v_2 = \underline{v}$. Comparing this case to the case in which $v_1 = v_2 = \underline{v}$, one can see that in the latter case there are additional types with an offer, $[v', \underline{v}]$, who sell in the first period. Because these types are lower than $\underline{v}$, the expected fundamental of a firm that hasn't sold in both periods is higher, that is, $E_2[v]\left(\underline{v}; v'\right) > \underline{v}$. This, in turn, will result in a equilibrium market value of $\hat{h}_2(v') > \underline{v}$.
   - (c) Now consider the case in which the sale threshold in period 1 is higher, $v_1 = v'' > \underline{v}$ and $v_2 = \underline{v}$. Firms of type $[\underline{v}, v'']$ sell only if they have an offer in period 2. Compared to the case in which $v_1 = v_2 = \underline{v}$, the type population who does not sell in both periods has additional types in $[\underline{v}, v'']$. these types are higher than $\underline{v}$, and thus $E_2[v]\left(\underline{v}; v''\right) > \underline{v}$. This, in turn, will result in an equilibrium market value of $\hat{h}_2(v'') > \underline{v}$.

   I conclude that $\hat{h}_2(v_1)$ is increasing if and only if $v_1 \geq \underline{v}$, and that in this region $\hat{h}_2(v_1) < v_1$.
3. From (A.6) and Proposition (1) property 1, we know that $\hat{h}_2(v_1^*) < v_1^*$. Thus, $v_1^* > \underline{v}$, and therefore $\frac{\hat{h}_2(v_1)}{\partial v_1}|_{v_1=v_1^*} > 0$. By Lemma 1, this entails $\frac{\partial v_2^*}{\partial v_1}|_{v_1=v_1^*} > 0$ and summarizes the proof of part 1.

Part 2 of the lemma is immediate given part 1 and Equation 6, because lower α is identical to a higher λ. ■

A.7 The effect of q on sale thresholds
A.7.1. Proof of Lemma 4.

Proof. I first prove the following lemma.

Lemma 14. $v_1^*(\alpha, q)$ and $v_2^*(\alpha, q)$ obtain

1. a global supremum at $q=0$. Specifically, $\lim_{q\to 0} v_1^* = \lim_{q\to 0} v_1^* = E_0[v]$; and
2. a global infimum at $q=1$. Specifically, $\lim_{q\to 1} v_1^* = \lim_{q\to 1} v_1^* = \underline{V}$.

Proof. I prove the lemma in several parts:

1. $v_2^*(\alpha, q)$ have a global supremum at $q=0$ and global infimum at $q=1$ – It is easy to see from Lemma 1 and equation (2) that $\lim_{q\to 0} v_2^* = E_1[v]$ and $\lim_{q\to 1} v_2^* = \underline{V}$. From (A.1) it is clear that, for any v_1^*, $\lim_{q\to 0} F_1(v) = F_0(v)$ and thus $\lim_{q\to 0} E_1[v] = E_0[v]$ (intuitively, as the probability of matching with a buyer approaches zero, the distribution of types that has not sold does not change). Thus, $\lim_{q\to 0} v_2^* = E_0[v]$, which is clearly a global supremum.

2. $v_1^*(\alpha,q)$ have a global supremum at $q=0$ – By Equation (5), $\lim_{q\to 0} h_1^* = h_2^*$, and thus by Equation (3) $\lim_{q\to 0} v_1^* = \lim_{q\to 0} v_2^* = E_0[v]$.
3. $v_1^*(\alpha,q)$ have a global infimum at $q=1$. Using Equation (5) we can write the limit of the market value in the first period as

$$\lim_{q\to 1} h_1^* = E_1\left[p_2(v) \mid v > v_2^*\right] = (1-\lambda)\chi \underline{V} + [1-(1-\lambda)\chi] E_1[v].$$

Now note that as $q \to 1$, the posterior distribution F_1 is truncated at v_1^* (all types above the threshold sell in the first period) and thus $\lim_{q\to 1} h_1^* < \lim_{q\to 1} E_1[v] < \lim_{q\to 1} v_1^*(\alpha,q)$. From Equation (3) it is clear that $\lim_{q\to 1} v_1^* = \underline{V}$. ■

Part 1 – Given part 2 of Lemma 14, it is clear that in each period $t=\{1,2\}$ there exists a threshold $\underline{q_t}$ such that $v_t^*(\alpha,q)$ is decreasing in q for any $q > \underline{q_t}$ and $v_t^*(\alpha,q) \geq v_t^*(\alpha,\underline{q_t})$ for any $q < \underline{q_t}$.

For part 2, first recall from Lemma 3 that $v_t^*(\alpha_l,q) - v_t^*(\alpha_h,q) > 0$ for all q. By lemma 14 (2), $\lim_{q\to 1} v_t^*(\alpha_l,q) - v_t^*(\alpha_h,q) = 0$. Thus, for any $\alpha_l < \alpha_h$ and q', one can find $\underline{q_t^\alpha}$ such that for any $q > \underline{q_t^\alpha}$, $v_t^*(\alpha_l,q) - v_t^*(\alpha_h,q)$ is decreasing in q and

$$v_t^*(\alpha_l,q') - v_t^*(\alpha_h,q') > v_t^*(\alpha_l,q) - v_t^*(\alpha_h,q).$$

Finally, let $\underline{q} = \max\left\{\underline{q_1}, \underline{q_2}, \underline{q_1^\alpha}, \underline{q_2^\alpha}\right\}$ to finalize the proof of the lemma. ■

A.7.2. Monotonicity of Thresholds with respect to *q*. Lemma 14 establishes that both thresholds obtain a global supremum at $q=0$ and a global infimum at $q=1$, but does not show that the threshold monotonically decrease in q. The following lemma shows that this is always the case if β or λ are low enough. The proof also shows that the equilibrium market values h_1^* and h_2^* are decreasing in q.

Lemma 15. There exist a constant $\underline{D} \in (0,1]$ such that, if $\beta\lambda < \underline{D}$ then $\frac{\partial v_t^*}{\partial q} < 0$ for $t \in \{1,2\}$ and any $q \in (0,1)$.

Proof. I prove the lemma in a series of steps:

1. $\frac{\partial \hat{h_2}}{\partial q} < 0$ and $\frac{\partial \hat{v_2}}{\partial q} < 0$ – Denote by $\hat{v_2}(F_0, v_1, q)$ and $\hat{h_2}(F_0, v_1, q)$ the equilibrium threshold and market value in period 2 for a given history, in the same way as in the proof of Proposition 3. From the analysis in Section 2 we know that $\hat{v_2}(F_0,v_1,q) = \hat{h_2}(F_0,v_1,q) = v^M(F_1(F_0,v_1),q)$. Observe from Equation (2) that $\frac{\partial v^M(F_1,q)}{\partial q} < 0$, and thus $\frac{\partial \hat{h_2}}{\partial q} < 0$ and $\frac{\partial \hat{v_2}}{\partial q} < 0$.
2. Let $x_1(v)$ denote the expected value at the end of the game of an asset of type v that hasn't been sold until the end of period 1. $x_1(v)$ takes into account the possibility of a sale in period 2 for a price below v. Given the analysis of Section 2, we can write

$$x_1(v; v_1, q, \lambda) = (1 - q(1-\lambda)\chi)v + q(1-\lambda)\chi \cdot \min\left\{\hat{v_2}(F_0, v_1, q), v\right\}.$$

Note that $x_1(v)$ is increasing in v and, because $\frac{\partial \hat{v_2}}{\partial q} < 0$ as found in step 1, then $\frac{\partial x_1(v)}{\partial q} < 0$ for all v.
3. $\frac{\partial \hat{h_1}}{\partial q} < 0$ – Denote by $\hat{h_1}(F_0, v_1, \lambda, q) \equiv E_1[x_1(v; v_1, q, \lambda); v_1, q, \lambda]$ the market value of a firm who hasn't sold at the first period as a function of the game history and parameters. Note that a change in q affects $\hat{h_1}$ through a change in the distribution F_1 and through a change in $x_1(v)$.

From Equation (A.1) one can see that for any $q' > q$ the distribution $F_1\left(v; v_1^*, q, F_0\right)$ FOSD $F_1\left(v; v_1^*, q', F_0\right)$, and thus $E_1[u(x)]$ is decreasing in q for any nondecreasing function $u(x)$. This, together with step 2, which established the effect of q on $x_1(v)$, entail that $\hat{h_1}(F_0, v_1, \lambda, q)$ is decreasing in q.

4. There exist $\underline{D}$ such that $\frac{\partial v_1^*}{\partial q}<0$ for $\lambda\beta<\underline{D}$ – using (A.6) to differentiate v_1^* and some algebra we obtain

$$\frac{\partial v_1^*}{\partial q}=\frac{\frac{\partial \hat{h}_1}{\partial q}+\beta(1-q\lambda)\frac{\partial \hat{h}_2}{\partial q}}{1+\beta(1-q\lambda)}+\frac{\beta\lambda\left(v_1^*-v_2^*\right)}{1+\beta(1-q\lambda)}$$

Given steps 1 and 3, the first argument of the RHS is negative; given Proposition (1) (1), the second argument is positive. Clearly, $\lim_{\beta\lambda\to 0}\frac{\partial v_1^*}{\partial q}<0$, and this is sufficient for the existence of $\underline{D}>0$ such that $\frac{\partial v_1^*}{\partial q}<0$ if $\beta\lambda<\underline{D}$.

5. If $\lambda\beta<\underline{D}$ then $\frac{\partial v_2^*}{\partial q}<0$ – by definition,

$$\frac{\partial v_2^*}{\partial q}=\frac{\partial \hat{v}_2(F_0,v_1,q)}{\partial q}\,|_{v_1=v_1^*}+\frac{\partial \hat{v}_2(F_0,v_1,q)}{\partial v_1}\,|_{v_1=v_1^*}\cdot\frac{\partial v_1^*}{\partial q}.$$

Step 1 establishes that $\frac{\partial \hat{v}_2}{\partial q}<0$ for all v_1^*. Step 2 in the proof of Lemma 3 establishes that $\frac{\partial \hat{v}_2(F_0,v_1,q)}{\partial v_1}\,|_{v_1=v_1^*}>0$. Thus, if $\frac{\partial v_1^*}{\partial q}<0$ then $\frac{\partial v_2^*}{\partial q}<0$ and, by step 4, if $\lambda\beta<\underline{D}$ then $\frac{\partial v_2^*}{\partial q}<0$. ■

A.7.3. A Model with Time-Varying q. This section briefly presents a model where q changes randomly over time. Consider the baseline model with a time varying q_t. The parameter q_t is determined and publicly revealed at the beginning of each period. For simplicity, assume that $q_t\in\left\{q^L,q^H\right\}$ with $q^L<q^H$. Market conditions change over time in a Markovian process, with a persistence probability of $\rho\equiv\Pr(q_2=q_1)\geq 0.5$ (some persistence in assumed). Denote $q_1^c\equiv\left\{q^L,q^H\right\}-q_1$ the complementary state to q_1, so that $\Pr\left(q_2=q_1^c\right)=1-\rho$. I consider only the case of no inventories ($\alpha=1$) but the model can be adapted for $\alpha<1$.

First, note that the analysis of the second and last periods is the same as that in the baseline model, with a threshold equilibrium of $v_2^*(q_2)=\hat{v}_2(F_0,v_1,q_2)=\hat{h}_2(F_0,v_1,q_2)=v^M(F_1(F_0,v_1,q_1),q_2)$ (Lemma 1). Step 1 of the proof of Lemma 15 entails that, for a given v_1, $h_2^*(q^L)>h_2^*(q^H)$.

Now consider the first period. It is easy to show that the equilibrium involves a threshold in this case – the outside option u_1^{NS} is weakly less sensitive to v_1 in this model compared to the baseline model. I now prove the following lemma regarding the properties of market values in this model

Lemma 16. In a model with time-varying q, market values have the following properties: (1) $h_1^*(q^L)>h_2^*(q^L)>h_2^*(q^H)$; (2) $h_1^*(q^H)>h_2^*(q^H)$; and (3) $h_1^*(q^L)\geq h_1^*(q^H)$.

Proof. Let $\hat{h}_1(F_0,v_1,\lambda,q)$ be the market value in the *baseline model* with a constant q, as defined in the proof of Proposition 3 and Equation (A.5). Because of the martingale nature of market values,

$$h_1^*(q_1)=\rho\hat{h}_1(F_0,v_1^*(q_1),\lambda,q_1)+(1-\rho)\hat{h}_1(F_0,v_1^*(q_1),\lambda,q_1^c). \tag{A.7}$$

Note that by step 3 of the proof of Lemma 15, $\hat{h}_1(F_0,v_1,\lambda,q^L)>\hat{h}_1(F_0,v_1,\lambda,q^H)$, and so

$$\hat{h}_1(F_0,v_1,\lambda,q^H)<h_1^*(q_1)<\hat{h}_1(F_0,v_1,\lambda,q^L).$$

Together with $\rho\geq 0.5$ this proves part (3). For parts (1) and (2) observe that, by Lemma 9, $\hat{h}_1(F_0,v_1,\lambda,q)>\hat{h}_2(F_0,v_1,q)=h_2^*(q)$. ■

I now turn to analyze v_1^*. The expected payoff of type v in period 2 in case she chooses not to sell and the probability is q_2 is

$$Eu_2(v,q_2) \equiv (1-q_2)u_2^{\mathrm{NS}}(v)+q_2 \cdot \max\left\{u_2^{\mathrm{NS}}(v), p_2(v)\right\}.$$

Substituting (1) and $u_2^{\mathrm{NS}}(v)=\chi h_2+(1-\chi)v$ we obtain

$$Eu_2\left(v_1^*,q_2\right)=(1-\chi)v_1^*+\chi\left[(1-q_2\lambda)h_2^*(q_2)+q_2\lambda\max\left\{h_2^*(q_2),v_1^*\right\}\right].$$

The expected continuation payoff of type v_1^* in period 1 in case she chooses not to sell is

$$U_1^{\mathrm{NS}}(v_1^*)=\chi h_1^*(q_1)+(1-\chi)v_1^*+\beta\left[\rho Eu_2\left(v_1^*,q_1\right)+(1-\lambda)Eu_2\left(v_1^*,q_1^c\right)\right],$$

and the indifference condition $v_1^*=u_1^{\mathrm{NS}}\left(v_1^*\right)\equiv\frac{U_1^{\mathrm{NS}}\left(v_1^*\right)}{1+\beta}$ results in

$$v_1^*(q_1)=\frac{h_1^*(q_1)}{1+\beta}+\frac{\beta}{1+\beta}\cdot\left\{\begin{array}{c}\rho\left((1-q_1\lambda)h_2^*(q_1)+q_1\lambda\max\left\{h_2^*(q_1),v_1^*\right\}\right)\\+(1-\rho)\left((1-q_1^c\lambda)h_2^*(q_1^c)+q_1^c\lambda\max\left\{h_2^*(q_1^c),v_1^*\right\}\right)\end{array}\right\}. \quad \text{(A.8)}$$

Lemma 17. $v_1^*(q_1)$ obtain a global infimum at $q=1$. Specifically, $\lim_{q_1\to 1}h_1^*(q_1)=\lim_{q_1\to 1}v_1^*(q_1)=\underline{V}$.

Proof. Assume, in contrast, that $\lim_{q\to 1}v_1^*(q)=v_1^*>\underline{V}$. Note that as $q\to 1$, the posterior distribution F_1 is truncated at v_1^* (all types above the threshold sell in the first period) and thus $\lim_{q\to 1}E_1[v](q_1)<\lim_{q\to 1}v_1^*(q)$. by definition, $h_1^*(q_1)<E_1[v](q_1)$ and $h_2^*(q_2)<E_1[v](q_1)$ for any q_2 and v_2^*. Thus, the limit of the RHS of (A.8) as $q_1\to 1$ is less than v_1^* – a contradiction. Thus $\lim_{q_1\to 1}v_1^*(q_1)=\lim_{q_1\to 1}h_1^*(q_1)=\lim_{q_1\to 1}\hat{h}_2(F_0,v_1^*(q_1),q_1)=\underline{V}$ ■

Given Lemma 17 it is clear that if q^H is high and close to one, then $v_1^*(q^L)>v_1^*(q^H)$. This is formalized in the following corollary.

Corollary 2. There exists a probability threshold $\underline{q_1}\in(0,1)$ such that, if $q^H>\underline{q}$ then $v_1^*(q^L)>v_1^*(q^H)$ for any $q^L<q^H$.

The corollary above establishes the amplification of negative demand shocks even in a model with time-varying q; a decrease in market conditions results in a greater decrease in volume due to the increase in thresholds, even though market conditions may recover in period 2.

A.8 Proof of Lemma 5

Proof. The proof of part 1, that is, that a sale threshold v_t^* exists in each period, is similar to the one in Proposition 2. Note that the threshold type obtains a price $p_t(v_t^*)=w(v_t^*)$ (see Equation (4) and the discussion that follows it), and thus always sells the maximal amount α^H.

For parts 1 and 2, note that, as explained in Section 5.2, the periodic payoff of the seller after a sale is $\alpha p_t(v)+(1-\alpha)v$ (because the sale discloses the value of the asset). From property 6 of Proposition 2 we know that in each period $\hat{v}_t<\overline{V}$ exists and that $v \lesseqqgtr p_t(v)$ if $v \lesseqqgtr \hat{v}_t$. Thus types $v\in[v_t^*,\hat{v}_t)$ maximize their payoff by choosing α_H, while types $v\geq\hat{v}_t$ maximize their payoff by choosing α_L. ■

A.9 Proof of Lemma 6

Proof. Consider the trading stage at period t and assume the market believes that $p_t(v)=v$. Thus, following a sale of asset of type v for price p, the buyer's payoff is

$$\chi \cdot p+(1-\chi)v-p=(1-\chi)(v-p).$$

The maximal price that a seller of type v can charge the buyer is therefore indeed $p=v$. Because the seller can obtain $p_t(v)=v$ if she chooses to sell in each period t, then we are back to the baseline model with $\lambda=1$, and therefore the equilibrium of the game is the same.

To see that there is no other equilibrium with a monotone $p_t(v)$, note first that in such a equilibrium $E\left[v \mid \text{Sale for price } p_t(\text{v})\right]=v$, and, thus, it cannot be that $p_t(v)>v$. Now consider an equilibrium with a monotone $p_t(v)$ where $p_t(v')<v'$ for a given type v' and period t. Because the type space is continuous, there exists a number $\epsilon>0$ such that $p_t(v')\leq v'-\epsilon$. If $p_t(v)$ is strictly increasing then $p(v'-\epsilon)<p(v)$, and a seller of type $v'-\epsilon$ can ask $p(v')$ for the asset and the buyer will accept (because $\chi \cdot v'+(1-\chi)(v'-\epsilon)>p(v')$) – a contradiction. If $p_t(v)$ is strictly decreasing then $p(v')<p(v'-\epsilon)\leq v'-\epsilon$, and thus a seller of type v' can offer the buyer $p(v'-\epsilon)$ (the buyer will accept because $\chi \cdot (v'-\epsilon)+(1-\chi)v'>p(v'-\epsilon)$) – a contradiction. ■

A.10 Proof of Lemma 8

Proof. In this model, the second (and last) period is identical to that of the baseline model, and the analysis is the same as in Section 2, leading to $v_2^*=v_2^M$. Recall also that $u_2^{\text{NS}}(v)=\chi \cdot v_2^*+(1-\chi)v$. In the first period, the seller's continuation payoff if she does not sell is

$$U_1^{\text{NS}}(v)=\begin{cases} v+\beta\left[\chi \cdot v_2^*+(1-\chi)v\right] & v\leq v_2^* \\ v+\beta\left[\chi\left(q\lambda v+(1-q\lambda)v_2^*\right)+(1-\chi)v\right] & v>v_2^*, \end{cases}$$

and thus, her outside option is

$$u_1^{\text{NS}}(v)=\begin{cases} \frac{(1+\beta(1-\chi))v+\beta\chi \cdot v_2^*}{1+\beta} & v\leq v_2^* \\ \frac{[1+\beta(1-\chi(1-q\lambda))]v+\beta\chi(1-q\lambda)v_2^*}{1+\beta} & v>v_2^*. \end{cases}$$

Note that $\frac{\beta(1-q\lambda)}{1+\beta}<\frac{\beta}{1+\beta}<1$, and therefore $u_1^{\text{NS}}(v)\lesseqgtr v_2^*$ iff $v\lesseqgtr v_2^*$. We have obtained $u_1^{\text{NS}}(v_2^*)=u_2^{\text{NS}}(v_2^*)=v_2^*$, and, thus, there is a threshold equilibrium in first period (part (1)) and $v_1^*=v_2^*$ (part (2)).

For part (3), define $\underset{\sim}{v}=v_1^*=v_2^*$ as the threshold of the game in both periods. Let $\hat{v}_2(v_1)$ be the equilibrium threshold in the second period that satisfies Lemma 1, as a function of the threshold in the first period. By definition, $\underset{\sim}{v}=\hat{v}_2(\underset{\sim}{v})$. As part of the proof of Lemma 3 (step 2), it is shown that $\underset{\sim}{v}$ exists, is unique, and $\underset{\sim}{v}=\min_{v_1^*}\hat{v}_2(v)$. Given that $v_1^*\geq v_2^*$, we obtain that both thresholds are minimized, and the probability of a sale is maximized. ■

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